Dynamical regime of electron transport in correlated one-dimensional conductor with defect

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Abstract

The electron transport in a 1D conductor with an isolated local defect such as an impurity or a non-adiabatic contact is studied theoretically. New regime of conduction in correlated 1D systems is predicted beyond the well-known regime of tunneling resulting in the power-law I-V-curves. In this regime a quantum wire becomes "opened" at voltage biases above the threshold value V_T, giving rise to a rapid increase of the dc current, $\bar{I}$, accompanied by ac oscillations of frequency $f = \bar{I}/e$. The effect of ac generation is related to sliding of the Friedel oscillations of electronic density produced by the defect. Manifestations of the effect resemble the Coulomb blockade and the Josephson effect. The spin bias applied to the system is shown to affect the I-V curves due to violation of the spin-charge separation at the defect site. In short quantum wires of length $L < L_0 \sim \hbar v_F / eV_T$ and at high temperatures $T > T_0 \sim eV_T/k_B$ the effect is destroyed by fluctuations. The threshold voltage V_T is determined by $2k_F$-component of impurity potential renormalized by fluctuations. The 1D conductor is described in terms of the Tomonaga-Luttinger Hamiltonian with short range or long-range Coulomb interaction by means of the bosonization technique. We derive boundary conditions that take into account relaxation in the leads and permit to solve non-equilibrium problems. Charge fluctuations are studied by means of Gaussian model which can be justified strictly in the limit of large voltages or strong enough inter-electronic repulsion. Spin fluctuations are taken into account strictly by means of the refermionization technique applicable in case of spin-rotation invariant interaction.

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I. INTRODUCTION

Basic electronic properties of three-dimensional (3D) solids are usually well described within the Landau's Fermi-liquid picture where low-energy excitations are single-electron quasiparticles that in many respects behave like non-interacting electrons. In the Fermi-liquid the interaction modifies energy spectrum of electrons and makes their life-time finite but still very long if the energy of electron is near the Fermi energy. This is not the case in 1D systems where interaction in some sense is always strong and cannot be considered as a small perturbation. The picture of single-electronic excitations fails in correlated 1D systems, and the only low energy excitations are bosonic charge and spin collective modes with a sound-wave-like spectrum. These excitations form the Luttinger liquid (LL) that is an alternative to the Fermi liquid for 1D electronic systems (for a review see Ref. 1 and 2). Such distinctive features of the LL as spin-charge separation and power-law suppression of tunneling into 1D systems have been confirmed experimentally, see e. g. Ref. 3. There are different realizations of 1D electronic systems demonstrating properties of the LL. The examples are semiconductor-based quantum wires in which dimensionality of the conduction electrons is reduced by dimensional quantization[4], metallic linear chains on Si surfaces[5], carbon nanotubes[6,7], conducting polymers[8], and edge states in FQHE, and 2D topological insulators[9]. There are also evidences of the LL-like effects in electronic transport in strongly anisotropic quasi-1D conductors[10,11] where the LL state can be stabilized by defects[12,13] or by formation of the charge-density wave (CDW) gap induced by electron-phonon coupling[14].

Inter-electronic interaction greatly affects electronic transport in conducting 1D systems. In particular, the back-scattering component of the impurity potential in 1D systems with repulsive inter-electronic interaction (interaction parameter $K_\rho < 1$) scales to infinity under renormalization group transformations. Hence, even isolated impurities form effectively large barriers and strongly suppress conductance[15–17]. This is reflected in power-law dependencies of conductance on voltage and/or temperature. This was confirmed experimentally in various 1D systems, including semiconductor quantum wires[4] and carbon nanotubes[6,7].

Strong effect of a single impurity on the electronic transport in 1D electron system is related to Friedel oscillations induced by impurity[1], and the power-law I-V curves were described in terms of macroscopic tunneling between different minima of a washboard potential tilted by external voltage bias. The periodic potential is associated with interaction between

smooth electronic density of the wire and Friedel oscillations. On the other hand, the limit of strong interaction between electrons in solids usually leads to the Wigner crystallization. But it is well-known that purely one-dimensional (1D) crystals cannot exist[18,19] because the long-range order is destroyed by fluctuations. So, strictly speaking, 1D Wigner crystals do not exist neither. However, in 1D gas with Coulomb repulsion density-density correlation functions contain oscillatory parts: the first one is at wave vector $2k_F$ and decays approximately as $1/x$, and the second one is at wave vector $4k_F$ and decays extremely slowly[20], like $e^{-c\sqrt{\ln x}}$, that is slower than any power-law. As the period corresponding to $4k_F$ oscillations is exactly the average inter-electron spacing, such a system can be considered as a 1D Wigner crystal with pseudo-long-range order[20]. In case of short range inter-electronic interaction (which takes place in gated quantum wires where the long-range part of the Coulomb interaction is screened by electrons in the metallic gate) the density correlations both at $2k_F$ and at $4k_F$ decay as power-law, with the exponent of $4k_F$ correlations being smaller. Despite the fact that there are oscillating terms in the correlation function, expectation values of these terms are absent in homogeneus LL as they are washed out by fluctuations. But the oscillating terms are present near defects since they strongly pin the LL and suppress density fuctuations. Sliding of the electronic crystals is known to contribute to conductivity of the system, the most studied case being quasi-1D CDW compounds[21]. As defects violate the space invariance they pin the CDW and for the CDW sliding the driving electric field must be larger than a threshold value. Above the threshold field the CDW starts to slide and to contribute to the current resulting in non-linear collective electron transport and generation of the so-called narrow-band noise at washboard frequencies[21] corresponding to a shift of the CDW by one period. As long as the LL can be interpreted as a 1D form of the 1D Wigner crystal, one can expect that the similar dynamic regime of depinning and sliding must appear in correlated 1D electron system as well. Thus, one can expect that at large enough voltages the system can roll out from the minimum of the washboard potential, namely, at voltages exceeding a threshold value determined by the slope of the washboard. Such a behavior is expected, at least, in the quasiclassical limit when quantum fluctuations at the impurity site are suppressed by strong interaction between electrons. This corresponds to very small values of the interaction parameter K_ρ in case of short-range interaction or to Coulomb interaction when effective K_ρ depends on momentum and is strongly suppressed in the long wave-length limit because of the long-range nature of the Coulomb interaction[1].

Such a scenario was addressed in our letter[22] where the dynamic regime of conduction in the LL was predicted for quasiclassical case of $K_\rho \ll 1$, in this regime the dc current is accompanied by oscillations with the washboard frequency $f = \bar{I}/e$.

Full I-V curves of a single-channel LL with a single impurity were studied by means of thermodynamic Bethe ansatz technique by Fendley et al[23]. Egger and Grabert[24] calculated the I-V curves for specific value of interaction parameter $K_\rho = 1/2$ using the refermionization technique which makes the Hamiltonian quadratic and, hence, solvable exactly. Calculation[25] of the current in the limit of strong interaction, $K_\rho \ll 1$, resulted in S-shaped I-V curves, and the authors concluded that by putting such a quantum wire into a properly designed load circuit, self-sustained current oscillations can be generated. These oscillations are a consequence of instability induced by negative differential conductivity, and their origin different from the mechanism discussed in Ref. 22 and considered here.

In all these papers the equilibrium distribution function of incident particles (non-interacting fermions, kinks and antikinks, etc) was assumed while distribution of the transmitted particles was not the equilibrium one. However, bosonic excitations of the LL are reflected from the leads even in case of adiabatic contacts because the interaction is present inside the wire and is absent in the leads. Reflection coefficient equals $r = \frac{1-K_\rho}{1+K_\rho}$ which approaches to unity in the limit $K_\rho \to 0$. As long as relaxation processes inside the wire are not taken into account in the Hamiltonian, the transmitted waves do not relax inside the wire even in the limit of very large length. Thus the incident waves must consist in part of the waves reflected from the contact. So distribution function of incident particles must not be necessarily the equilibrium one, and this applies equally to fermions derived from bosons after the refermionization. In this case one needs to take into account relaxation processes induced by coupling of the quantum wire to 2D or 3D Fermi liquid of the leads which can be considered as a heat bath. This can be made by means of boundary conditions. They allow to find non-equilibrium fluctuations inside the wire in terms of equilibrium density fluctuations in the leads. One of our goals is to include such effects into study of the electronic transport in quantum wires. This will be performed using boundary conditions for operators which generalize the radiative boundary conditions derived by Egger and Grabert[24] for expectation values. Furthermore, using boundary conditions for non-ideal contacts, we demonstrate that Friedel oscillations which appear near non-adiabatic contact result in non-stationary effects similar to those of impurities. Note that we consider non-

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ideal contacts in which backscattering is induced by the geometry, and this is different from the model of Ref. 26, in which backscattering was derived in a microscopic lattice model with a spatially dependent nearest-neighbor interaction turned on near the contact from zero to a bulk value. In the local LL model characterized by a spatial dependence of the LL parameters the backscattering does not appear[26–29].

The dynamic regime of conduction was considered in Ref. 22 in the limit of strong inter-electronic repulsion, when fluctuations at the impurity site are small. The results were also limited by the case of very long conducting channels, longer than the damping length of excitations due to coupling of electrons inside the wire to a dissipative bosonic bath (phonons, density fluctuations in a metallic gate, and so on) when one can neglect that current pulses generated at the impurity reflect from the contacts. Here we consider conduction in more realistic case of shorter conducting channel when fluctuations in the channel are governed by boundary conditions. We study solutions at various values of the applied voltage concentrating on the limit of large voltages/currents, where Gaussian fluctuations dominate, and non-Gaussian part of fluctuations is smaller. This enables us to calculate I-V curves, the amplitude of the ac current and noise spectrum more strictly. The I-V curves that we found differ from the results of Ref. 23 and 24. In addition, we consider both a single mode (spin-polarized) and a spinful 1D electronic systems. The interaction between electrons is assumed to be strong enough and of two types, either a short range characterized by a constant interaction parameter $K_\rho < 1$, or the long range Coulomb interaction. The short-range interaction describes gated quantum wires where the long-range part of the interaction is screened by electrons of 3D gate electrodes.

The structure of the paper is as follows. In Sec. II we formulate the problem, and derive boundary conditions for equations of motion for Heisenberg operators of the displacement fields of the LL. Using the boundary conditions we derive equations of motion for the displacement field at the impurity site. These equations resemble equation of motion of a quantum pendulum or equation for the superconducting phase in the Josephson junctions. In Sec. III we use our equations to study electronic transport in spinless LL. Using the Gaussian model to account for fluctuations, we calculate I-V curves in several limits, analyze noise spectrum, study non-Gaussian corrections, and finally discuss validity of the Gaussian approximation. In Sec. IV we consider the spinful LL with strong enough interaction between electrons (either very strong short-range or moderate long-range Coulomb

interactions). This is the limit in which charge fluctuations at the defect are small. Though spin fluctuations are still large, they are taken into account strictly by means of refermionization method in spin sector. This method works in case of spin-rotation invariant interaction ($K_\sigma = 1$, interaction does not depend on spin). In Sec. V we show that non-ideal contacts induce non-stationary effects similar to those induced by impurities. In Sec. VI we formulate conclusions.

Below we set e, $\hbar$ and k_B to unity, restoring dimensional units in final expressions when necessary.

II.   GENERAL FORMULATION

A.   Problem formulation

We consider a correlated 1D conductor containing a defect located at $x = 0$, and adiabatically connected to ideal Fermi-liquid reservoirs at $x = \pm L/2$, or an ideal 1D conductor without impurity connected to reservoirs by non-adiabatic contacts. The Hamiltonian of the system with impurity consists of two terms

$$H = H_0 + H_i.$$

The first one is the bosonised Tomonaga-Luttinger (TL) Hamiltonian that maps the 1D system of interacting electrons to free massless bosons described in terms of the displacement fields $\hat{\Phi}_\rho(t, x)$ and $\hat{\Phi}_\sigma(t, x)$ and the conjugated fields $\hat{\Pi}_\nu(t, x) = \partial_x \hat{\Theta}_\nu / \pi$ describing the momentum density, $[\hat{\Phi}_\nu(t, x), \hat{\Pi}_{\nu'}(t, y)] = i\delta(x - y)\delta_{\nu,\nu'}$. Here $\nu = \rho, \sigma$ denotes charge and spin channels, correspondingly. These fields are related to fermionic electron operators for right ($r = +1$) and left ($r = -1$) moving electrons with spin s

$$\hat{\psi}_{rs}(x) = \lim_{\alpha \to 0} \frac{e^{irk_F x}}{\sqrt{2\pi\alpha}} \hat{\eta}_{r,s} e^{-i\frac{1}{\sqrt{2}}[\hat{\Theta}_\rho(x) - r\hat{\Phi}_\rho(x) + s(\hat{\Theta}_\sigma(x) - r\hat{\Phi}_\sigma(x))]}, \tag{1}$$

where $\hat{\eta}_{r,s}$ are ladder operators, and $\alpha \sim k_F^{-1}$ is the small cut-off length of the Tomonaga-Luttinger model. The standard TL Hamiltonian in the Fourier transformed form reads[1,2]

$$H_0 = \frac{\pi v_F}{2} \sum_{\nu = \rho, \sigma} \int \frac{dq}{2\pi} \left\{ \hat{\Pi}_\nu^2 + \frac{1}{\pi^2 K_\nu^2} q^2 \hat{\Phi}_\nu^2 \right\}. \tag{2}$$

Here the LL parameters K_ν, playing the role of the stiffness coefficients of the elastic string described by Hamiltonian (2), are related to the electron-electron interaction potential, and

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measure the strength of interaction between electrons. In the spin-rotation invariant case considered in our study, $K_\sigma = 1$, and

$$K_\rho(q) = \frac{1}{\sqrt{1 + \frac{g(q)}{\pi v_F}}}. \tag{3}$$

where $g(q)$ is the Fourier transformed interaction potential. In case of infinite 1D gas with long-range Coulomb interaction described by the approximate form $V_C(x) = \frac{e^2}{\epsilon\sqrt{x^2+d^2}}$, where ϵ is a background dielectric constant and d is a diameter of quantum wire, one obtains $g(q) = 2\frac{e^2}{\epsilon}K_0(|qd|)]$. Thus,

$$K_\rho(q) = \frac{1}{\sqrt{1 + \gamma K_0(|qd|)}}, \quad \gamma = \frac{2e^2}{\pi\hbar v_F\epsilon} = \frac{2}{137\pi}\left(\frac{c}{v_F}\right)\frac{1}{\epsilon}, \tag{4}$$

where γ is dimensionless parameter which measures the strength of the Coulomb repulsion between the electrons.

This case was described by Schulz[20] as a "1D Wigner crystal" because the system exhibits a $4k_F$ electron density modulation characterized by extremely slow (slower than any power law) decay of the density correlations, which is induced by long-range nature of Coulomb interaction.

In case of the short-range interaction the dependence of g on wave-vector q is usually neglected, then $K_\rho < 1$ corresponds to repulsive and $K_\rho > 1$ to attractive interaction. The model with short-range interaction is applicable to gated quantum wires where the long-range part of interaction is screened by 3D gate electrodes. For quantum wires one can roughly estimate the value of K_ρ within a simple model of the nanowire of diameter d located at distance d_1 from the metallic gate

$$K_\rho^{-2} \approx 1 + \frac{2e^2}{\pi\hbar v_F\epsilon}\ln\frac{d+d_1}{d} = 1 + \frac{1}{137}\left(\frac{c}{v_F}\right)\frac{2}{\pi\epsilon}\ln\frac{d+d_1}{d}.$$

Eigenmodes of free electronic system described by Eq. 2 are bosonic waves with a sound-like spectrum, $\omega_\nu(q) = qv_\nu(q)$ where $v_\nu(q) = v_F/K_\nu(q)$ is the velocity of charge ($\nu = \rho$) and spin ($\nu = \sigma$) excitations.

In case of the long-range interaction and finite length of the system located at $|x| < L/2$ we have to modify the Coulomb potential in order to take into account screening of the interaction potential by metallic leads. The exact form of the screening depends on the geometry of the system. We will consider 3D metallic leads forming sheets of a plane

capacitor connected by the quantum wire. Then the screening by the leads can be depicted in terms of the image charges, and the interaction potential between charges located at x and x' is described as

$$V(x, x') = \sum_{n=-\infty}^{\infty} \left[V_C(x - x' + 2nL) - V_C(x + x' + 2nL + L) \right], \tag{5}$$

where the term with $n = 0$ describes the direct Coulomb interaction, and other terms are induced by image charges. Its contribution to the $\nu = \rho$ term in the Hamiltonian (2) in the coordinate representation reads

$$\int dx dx' \left\{ \partial_x \hat{\Phi}_\rho(x) V(x, x') \partial_{x'} \hat{\Phi}_\rho(x') \right\} \tag{6}$$

Since the operator of the particle density is given by expression $\hat{\rho} = -(\sqrt{2}/\pi)\partial_x \hat{\Phi}(x)$, this term has rather transparent physical meaning.

Interaction with the impurity is described in terms of the phases field $\hat{\Phi}_\nu(t, x)$ at the impurity position $x = 0$[1,2]

$$\hat{H}_i = - \int dx \frac{1}{\pi} W \delta(x) \cos \sqrt{2} \hat{\Phi}_\rho \cos \sqrt{2} \hat{\Phi}_\sigma, \tag{7}$$

where the impurity strength W is related to the back-scattering part of the impurity potential $U_i(x)$

$$W = \int \frac{dx}{\alpha} e^{2ik_F x} U_i(x).$$

The forward scattering is not included because it can be eliminated from the problem by redefinition of the field $\hat{\Phi}_\rho$[1]. Note that the form (7) of the impurity Hamiltonian follows from bosonization relations (1) and is related to $2k_F$-components of electron density. The Luttinger model does not contain higher harmonics, but in more general model the higher harmonics are present[2]. For example, for a model with non-linear electron dispersion, more general bosonization relations are used[30], and the higher harmonics do appear. However, such harmonics do not result in qualitative effects in electronic transport studied here.

Current and charge density perturbations in the system can be calculated in terms of $\hat{\Phi}_\rho$ by means of thermodynamic averaging of the expressions for corresponding operators

$$\hat{I} = \frac{\sqrt{2}}{\pi} \partial_t \hat{\Phi}_\rho, \quad \hat{\rho} = -\frac{\sqrt{2}}{\pi} \partial_x \hat{\Phi}_\rho. \tag{8}$$

Further, the displacement field determines also the operator of $2k_F$-component of charge density perturbations

$$\hat{\rho}_{2k_F} = \frac{ek_F}{\pi} \cos(2k_F x + \sqrt{2} \hat{\Phi}_\rho) \cos \sqrt{2} \Phi_\sigma. \tag{9}$$

In pure infinite 1D electronic system fluctuations of the phase field $\hat{\Phi}$ are infinite, $\langle\hat{\Phi}^2\rangle = \infty$, therefore, the expectation value of the right-hand side of Eq. (9) and, hence, $2k_F$-component of charge density is zero. However, fluctuations of $\hat{\Phi}$ can be finite at the impurity position, and this results in Friedel oscillations, i. e., in $2k_F$ modulation of charge density, $\rho_{2k_F} = \langle\hat{\rho}_{2k_F}\rangle$. The amplitude of oscillations decays with the distance. The larger electronic repulsion is, the slower Friedel oscillations decay. According to Eq. (8), the current flow is related to time variations of $\hat{\Phi}$, therefore, according to Eq. (9) the current passing the impurity implies a space shift of the Friedel oscillations by the current. The drag of the Friedel oscillations by current illustrates the origin of strong suppression of conductance by impurities and its dependence on inter-electronic interaction.

According to Eq. (8) the macroscopic current in the system can be expressed in terms of the expectation value of the displacement field. The latter can be found from equation of motion for the Heisenberg operator $\hat{\Phi}_\rho(t, x)$. Commuting $\hat{\Phi}_\rho$ with the Hamiltonian for the case of short range interaction we find

$$\left(v_\rho^2\partial_x^2 - \partial_t^2\right)\hat{\Phi}_\rho(t, x) = \sqrt{2}\pi v_F W \sin\sqrt{2}\hat{\Phi}_\rho(t, x)\cos\sqrt{2}\Phi_\sigma(t, x)\delta(x), \qquad (10)$$

where $v_\rho(q) = v_F/K_\rho(q)$ is, again, the velocity of charge (plasmonic) excitations. Equation of motion for the spin field has similar form, it can be obtained from (10) by substitution subscripts ρ by σ and vice versa.

At contacts we apply the boundary conditions for the operator $\hat{\Phi}$, which takes into account injection of electrons induced by external bias and relaxation processes induced by coupling of the quantum wire to 2D or 3D Fermi liquid in the current leads. The issue on the boundary conditions is considered in details in the next subsection.

B.   Boundary conditions

Boundary conditions for the single mode (spinless or spin-polarized) wire contacting with a 2D or 3D leads were derived in Ref. 33. Here we generalize this result for the spinful case. In order to derive the boundary conditions we use the ideas of the scattering approach (for a review see Ref. 34) and consider the region of the wire at $|x| < L/2$ as a scatterer.

Without the loss of generality we assume that longitudinal (along the x-axis) and transverse motions are separable. The longitudinal motion in the leads is characterized by wave

vectors k, spin s and energy $\varepsilon_l = \frac{k^2}{2m}$. The transverse motion is described by energy ε_n, the total energy being $\varepsilon = \varepsilon_l + \varepsilon_n$, where n is an index labeling transverse energies. We assume that electrons in the leads do not interact. Then we solve an equation of motion for electronic field operators in the leads using the continuity of both the field operators and their derivatives at $|x| = L/2$. This allows us to express the solution for the n-th transverse eigenstate in terms of the field operator $\hat{\psi}_b$ at the boundary. Since the results are very similar for both contacts we concentrate on the left lead, and the result for the right one will be given without derivation. For the contact at $x = -L/2$ we obtain

$$\hat{\psi}(x) = \hat{\psi}_b \cos kx + \frac{1}{k}\partial_x\hat{\psi}_b \sin kx. \tag{11}$$

This expression contains both incident and outgoing waves. According to the causality principle, the incident wave $\hat{\psi}_{in}(x)$ is determined by a state of the lead far away from the barrier. Therefore, $\hat{\psi}_{in}(x)$ must not depend on properties of the barrier. Equating the incident part of Eq. (11) to the form describing free particles we find

$$\hat{\psi}_b - \frac{i}{k_l}\partial_x\hat{\psi}_b = \frac{4\pi}{\sqrt{L}}\sum_{k>0}\hat{c}_{n,s,k}\delta(\varepsilon - \varepsilon_n - \frac{k^2}{2m}), \tag{12}$$

where $k_l = \sqrt{2m(\varepsilon - \varepsilon_n)}$ and $\hat{c}_{n,s,k}$ is an annihilation operator of an electron in the lead with a longitudinal momentum k and spin s in the n-th transverse mode of the lead.

Eq. (12) relates the field operator at the boundary to the equilibrium states of the n-th transverse mode in the lead. We are interested in finding a relation between the boundary value of the field operator corresponding to the lowest transverse eigenstate of the conducting wire and the incident state of the lead. To find this relation, we project Eq. (12) onto the eigenstates of the wire. Since transverse states of the lead are not eigenstates of the wire, we obtain an infinite system of linear equations for boundary values of the field operators $\hat{\psi}_j$ describing different transverse eigenstates j of the wire

$$\hat{\psi}_j - \sum_{j'}r_{jj'}\partial_x\hat{\psi}_{j'} = \hat{Z}_j, \tag{13}$$

where coefficients $r_{jj'}$ depend on the properties of the contact, and we do not calculate them as we do not need their explicit form. We must find a solution of Eqs. (13-12) for the state $j = 0$ describing the lowest subband which is responsible for an electronic transport in the wire, while the states $j > 0$ with higher transverse energies do not contribute to the

transport. It follows from Eq. (13) that the relation we are looking for has a form

$$A(\varepsilon)\hat{\psi}_0 + B(\varepsilon)\partial_x\hat{\psi}_0 = \frac{1}{\sqrt{V}} \sum_{\mathbf{n}=n,k>0} \gamma(k)\hat{c}_\mathbf{n} 2\pi\delta(\varepsilon - \varepsilon_\mathbf{n}), \qquad (14)$$

where the exact expressions for the coefficients in Eq. (14) depend on the shape of the contacts. The boundary condition for the right contact has the same form but with complex-conjugate coefficients.

Coefficients in Eq. (14) are not arbitrary. In particular, they must provide correct anti-commutation relations for electronic field operators. It is worthwhile to relate the coefficients to such physical parameters of the system as transmission probability t of incident electrons for non-interacting system. Therefore, we consider non-interacting electrons, for which we can easily solve the equations for the field operators inside the wire. Then we impose a re-quirement of fulfillment of anticommutation relations, and calculate the conductance. This allows us to reduce the number of undetermined constants. As it is more convenient to express boundary conditions in terms of physical values, we multiply Eq. (14) on the left by its Hermitian conjugate, then we transform the obtained equation to the time representation assuming that the coefficients are slowly varying functions of energy in the region close to the Fermi energy, and neglecting terms corresponding to transitions to upper empty energy bands of the wire. Finally, we find boundary condition for the left (right) contact for each spin direction

$$\frac{v_F}{t}\hat{\rho} \pm \hat{j} + v_F f\hat{\rho}_F = \frac{1}{V} \sum_{\mathbf{n},\mathbf{n}'} \hat{c}_{\mathbf{n}'}^+ \hat{c}_\mathbf{n} e^{i(\varepsilon_{\mathbf{n}'} - \varepsilon_\mathbf{n})t}, \qquad (15)$$

where $\hat{j}$ and $\hat{\rho}$ are operators of current and of the smooth part of charge density perturba-tions, $\hat{\rho}_F$ is the $2k_F$-component of charge density, which is related to the Friedel oscillations, f is a number of the order unity if the transmission probability is not close to unity, and $f \simeq \sqrt{2(1-t)}$ if $1 - t \ll 1$. Thus the Friedel oscillations disappear if the contacts are adiabatic.

In order to check the validity of conditions (15), we considered a wire with non-interacting 1D electrons which is attached to smoothly widening leads, so that the contacts are nearly adiabatic. We also assumed that there might be a potential step of the height $U_0 \ll \varepsilon_F$ at the interface. Under these assumptions we were able to use the quasiclassical approximation in the lead and match the quasiclassical solution outside the 1D conductor with the exact solution inside the channel. We found that the condition (15) yielded a correct result for

the conductance $G = tG_0$ in agreement with the Landauer formula, where $G_0 = e^2/h$ is the conductance quantum.

As we need the boundary conditions for the LL described in the bosonic representation we have to present (15) in the bosonised form. We should note that since one assumes a linear dispersion of electrons within the LL approach, the theory is valid provided that all energies are small in comparison with the Fermi energy. However, the amplitude of the term $v_F f \hat{\rho}_F$ which is responsible for the Friedel oscillations is of the order of the Fermi energy if $f = 2\sqrt{1-t}$ is not small. Therefore, we consider nearly adiabatic contacts with $\sqrt{1-t} \ll 1$, and neglect terms of the higher order in f.

Then we transform in a standard way[2] the fermionic operators to charge and spin density variables for right $(r = +1)$ and left $(r = -1)$ moving electrons

$$\rho_r = \frac{1}{\sqrt{2}}[\rho_{r,\uparrow} + \rho_{r,\downarrow}], \quad \sigma_r = \frac{1}{\sqrt{2}}[\rho_{r,\uparrow} - \rho_{r,\downarrow}]$$

and obtain the boundary conditions for bosonic field $\hat{\Phi}_\rho$ at the left and right contacts

$$v_F \partial_x \hat{\Phi}_\rho \mp \partial_t \hat{\Phi}_\rho + \sqrt{2} f \varepsilon_F \sin(\sqrt{2}\hat{\Phi}_\rho \mp k_F L) \cos\sqrt{2}\hat{\Phi}_\sigma = \hat{P}_\rho^{L,R} \tag{16}$$

$$v_F \partial_x \hat{\Phi}_\sigma \mp \partial_t \hat{\Phi}_\sigma + \sqrt{2} f \varepsilon_F \cos(\sqrt{2}\hat{\Phi}_\rho \mp k_F L) \sin\sqrt{2}\hat{\Phi}_\sigma = \hat{P}_\sigma^{L,R}, \tag{17}$$

where $\hat{P}_\nu^{L,R} = 2\pi v_F \hat{N}_\nu^{L,R}$, $N_\nu^{L,R}$ is the operators of excess number of charge $(\nu = \rho)$ and spin $(\nu = \sigma)$ densities in the left (L) and right (R) leads, respectively. The expectation values of the operators $P_\nu^{L,R}$ and correlation functions of their fluctuating parts $\delta\hat{P}_\nu^{L,R} = \hat{P}_\nu^{L,R} - \langle\hat{P}_\nu^{L,R}\rangle$ can be calculated easily from the right-hand part of Eq. (15). The average of $P_\rho^{L,R}$ for charge channel is proportional to the potential $U_\rho^{L,R}$ applied to the left (right) contact and, similarly, the expectation values $\langle\hat{N}_\sigma^{L,R}\rangle$ are equal to the excess spin densities in the leads, hence the difference of the expectation values $\langle\hat{P}_\sigma^R - \hat{P}_\sigma^L\rangle$ can be related to "spin bias" V_σ:

$$\langle P_\rho^{L,R}\rangle = U_\rho^{L,R}/\sqrt{2}, \quad \langle P_\sigma^{L,R}\rangle = U_\sigma^{L,R}/\sqrt{2},$$

Correlation functions are identical for both channels and for both contacts, while correlations between left and right contacts and between charge and spin operators are absent. In the frequency representation correlation functions for anti-commutators and commutators read

$$\langle\{\delta\hat{P}(\omega), \delta\hat{P}(\omega')\}\rangle = 4\pi^2\omega \coth\frac{\omega}{2T}\delta(\omega+\omega'), \quad \langle[\delta\hat{P}(\omega), \delta\hat{P}(\omega')]\rangle = 4\pi^2\omega\delta(\omega+\omega'). \tag{18}$$

If there is a metallic gate near the quantum wire we must take into account screening by the gate. Following the approach of Ref. 24 we find that the screening by the gate results in a modification of the factor in the first term of Eq. (16). Then the boundary conditions for the case of short-range interaction acquire the form

$$\frac{v_F}{K_\rho^2}\partial_x\hat{\Phi}_\rho \mp \partial_t\hat{\Phi}_\rho + \sqrt{2}f\varepsilon_F\sin(\sqrt{2}\hat{\Phi}_\rho \mp k_F L)\cos\sqrt{2}\hat{\Phi}_\sigma = \hat{P}_\rho^{L,R}. \qquad (19)$$

This modification of the factor before the spatial derivative can be also illustrated by means of the simple model in which the factor K_ρ is equal to 1 at the non-interacting lead $x = -L/2 - 0$ and step-like reaches its value in the wire at $x = -L/2 + 0$. Then we integrate the equation of motion (10) from $x = -L/2 - 0$ to $x = -L/2 + 0$ and assume that $\hat{\Phi}_\rho$ is a smooth function of x we obtain

$$\partial_x\hat{\Phi}_\rho(-L/2 - 0) = \frac{1}{K_\rho^2}\partial_x\hat{\Phi}_\rho(-L/2 + 0),$$

which explain transition from Eq. (16) to (19).

In case of a wire adiabatically connected to ideal Fermi-liquid reservoirs at $x = \pm L/2$ the boundary conditions reduce to

$$\left(\frac{v_F}{K_\nu^2}\partial_x \mp \partial_t\right)\hat{\Phi}_\nu(x = \pm L/2) = \hat{P}_\nu^{L,R}. \qquad (20)$$

Thermodynamically averaged boundary condition Eq. 20 for the charge channel is reduced to that derived in Ref. 24.

It looks natural that in case of gated quantum wire the gate screens externally applied electric field and the problem is described in terms of boundary conditions, as it was discussed in Ref. 24. Of course, inside the wire there is also an electric field induced by non-uniform distribution of electrons, but this electric field is taken into account by the interaction between electrons. However, it looks less clear whether one can describe the driving voltage by boundary conditions when there is no gate (the case of long-range interaction). Therefore, in case of long-range Coulomb interaction we considered two approaches. First, we derived equations of motion for the phase fields with driving dc voltage taken into account by boundary conditions. Second, we inserted the driving dc electric field into the Hamiltonian, when the external field appears in the equation of motion for the displacement field $\hat{\Phi}_\rho(x,t)$. But the equation of motion for the displacement field $\hat{\Phi}_\rho(t)$ at the impurity site turned out to be the same in both cases. For the external ac electric field the result of these two

approaches is different, and this is not surprising, because the ac-field can be induced by incident electromagnetic wave, and this could lead to results different from the case of the ac voltage applied to the contacts.

The fluctuating part of the boundary conditions takes into account that the leads play a role of a heat bath. Thus it describes relaxation in quantum wires induced by interaction with bulk leads. Such mechanism of relaxation is dominant in practically important case of the quantum wire which is shorter than the relaxation length related to coupling of electrons to phonons or to other dissipative bosonic bath.

Let us shortly discuss the role of damping of the plasmonic excitations. The equation of motion (10) does not contain damping. If one takes into account relaxation inside the wire then a finite damping must appear. Damping due to electrostatic coupling to metallic gate was studied in Ref. 31 and its effect on electronic transport passing an impurity was considered in Ref. 32. In Ref. 32 the effective action was reduced to a simple form that allows to find the Green's function of the system with finite damping. In this case the Fourier transformed retarded Green's function acquires the form

$$D_0^R = \frac{\pi v_F}{\omega^2 + i\omega\nu - q^2 v_\rho^2} \tag{21}$$

where ν is the damping constant. According to Ref. 31, coupling to fluctuations in the gate is relevant only at $K_\rho < 1/2$. However, following Ref. 32, one can consider $\nu > 0$ at all values of K_ρ as a phenomenological parameter bearing in mind that other sources of dissipation are possible. Below we concentrate on the case of not very long conducting channel, $L \ll l_\nu$, that is on the case which is opposite to that considered in our letter[22]. Then the damping constant ν can be neglected. However, even under condition $\nu = 0$ some dissipation in the wire is still present since the boundary conditions (20) with (18) connect the wire to a thermal bath. This can be demonstrated by calculation of the eigenmodes in a pure wire without impurity. From Eq. (10) with boundary conditions (20) we find the modes with spectrum $\omega_n = q_n v_\rho$ and eigenfrequencies given by relation

$$\omega_n = n\frac{\pi v_\rho}{L} - \frac{iv_\rho}{L}\ln\frac{1 + K_\rho^2}{1 - K_\rho^2}.$$

Similarly, damping of the order of v_F/L is present in case of the long-range interaction. We see that damping of the eigenmodes is finite, and the damping constant decreases when the length of the wire increases. Further, distribution function of the excitations in the wire is

determined by correlation functions (18) so that in the equilibrium state it is described by the Planck distribution. In the non-equilibrium state the excess energy can be dissipated in massive leads and if the wire is short enough this restricts heating effects.

C.   Equations of motion of the displacement field at the impurity site

In this section we consider ideal adiabatic contacts and derive equations of motion for the phases $\hat{\Phi}_\rho$ and $\hat{\Phi}_\sigma$ at the impurity. Consider first the case of short-range interaction. In order to solve equation of motion (10) for the phase field we use Fourier transformation with respect to time and formally solve differential equations for $\hat{\Phi}_\nu(\omega, x)$. Solutions are linear combinations of exponents $\exp\left(\frac{i\omega x}{v}\right)$ and $\exp\left(-\frac{i\omega x}{v}\right)$ with different coefficients to the left and to the right of impurity. We match the solutions at the impurity site and use the boundary conditions (20). In this way we express operators $\hat{\Phi}_\nu(\omega, x)$ in terms of their values at the impurity site, $x = 0$, and after inverse Fourier transformation obtain equations of motion for the displacement field at the impurity site. The equations read

$$\partial_t \hat{\Phi}_\rho + \frac{W}{\sqrt{2}} Z \otimes \sin \sqrt{2}\hat{\Phi}_\rho \cos \sqrt{2}\hat{\Phi}_\sigma = F \otimes \hat{P}_\rho. \tag{22}$$

$$\partial_t \hat{\Phi}_\sigma + \frac{W}{\sqrt{2}} \sin \sqrt{2}\hat{\Phi}_\sigma \cos \sqrt{2}\hat{\Phi}_\rho = \hat{P}_\sigma(t - t_L). \tag{23}$$

Here $\otimes$ means convolution in time, $P_\alpha = \hat{P}_\alpha^R - \hat{P}_\alpha^L$, and $Z(t)$ and $F(t)$ are defined by means of Fourier components

$$Z(\omega) = K_\rho \frac{1 - iK_\rho \tan \omega t_L}{K_\rho - i \tan \omega t_L}, \quad F(\omega) = \frac{K_\rho}{K_\rho \cos \omega t_L - i \sin \omega t_L}, \quad t_L = \frac{LK_\alpha}{2v_F}. \tag{24}$$

These equations resemble equations of motion of two coupled overdamped pendulums with some retardation function in the gravity term. Oscillatory dependence of $Z(\omega)$ and $F(\omega)$ describes multiple reflections of the bosonic excitations of the LL from contacts. This statement can be illustrated by the expression for Z in time representation

$$Z(t) = K_\rho \left[\delta(t) + 2 \sum_{m=1}^{\infty} r^m \delta(t - mt_L) \right], \quad r = \frac{1 - K_\rho}{1 + K_\rho}, \tag{25}$$

where r is the reflection coefficient of plasma excitations from the boundary between regions with different interaction constants. Eq. (25) is symmetric with respect to left and right contacts because we consider the impurity located in the middle of the conducting

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channel. Generalization to the case when impurity position is shifted from the middle is straightforward and we do not consider this case.

Consider now the case of long-range Coulomb interaction between the electrons. If we consider interaction potential in the system of finite length (5) formally at all values of coordinates, then we find that it is symmetric with respect to the left contact and periodic with period $2L$. This means that we can expand the field operators in Fourier series in the interval $(-L/2, L/2)$

$$\hat{\Phi}_n(x) = \frac{\hat{\Phi}_0}{2} + \sum_{n=0} \hat{\Phi}_n \cos q_n \left(x + \frac{L}{2} \right). \tag{26}$$

This result in simple and easily soluble equation of motion

$$(\omega^2 - q_n^2 v_n^2)\hat{\Phi}_n - v_F^2 \frac{2}{L}[\partial_x \hat{\Phi}(0) - (-1)^n \partial_x \hat{\Phi}(L)] = \frac{2\sqrt{2}v_F}{L} W\{\sin \sqrt{2}\hat{\Phi}_\rho(t,0) \cos \sqrt{2}\hat{\Phi}_\sigma(t,0)\}_\omega. \tag{27}$$

Then using the boundary conditions we derive equations of motion for the field operators at the impurity position $x = 0$, and find that they are similar to (22-23) but with different memory functions F and Z

$$Z(\omega) = \frac{i\omega R_+ - 2\omega^2(R_+^2 - R_-^2)}{1 + 2i\omega R_+}, \; F(\omega) = \frac{i\omega R_-}{1 + 2i\omega R_+}, \quad R_\pm(\omega) = \frac{v_F}{L} \sum_{k=-\infty}^{\infty} \frac{(\pm 1)^k}{\omega^2 - q_{2k}^2 v^2(q_{2k})}$$

with $q_n = \frac{\pi n}{L}$, $v^2 = v_F^2[1+2\gamma \mathrm{K}_0(|qd|)]$, where McDonald function describes Fourier transform of Coulomb interaction in a 1D conductor of transverse dimension d. We will concentrate on the case of long enough wires, and we will need functions Z and F at $\omega = 0$, when $Z(0) = F(0) = 1/2$, and at $\omega \gg v_F/L$, when summation can be approximated by integration. In the latter case we find with logarithmic accuracy

$$Z(\omega) \approx \frac{1}{2\sqrt{\gamma \ln \frac{2v_F}{\omega d}}}, \quad F(\omega) \approx \frac{e^{\frac{iq_\omega L}{2}}}{2\sqrt{\gamma \ln \frac{2v_F}{\omega d}}}, \quad q_\omega = \frac{\omega}{v_F\sqrt{2\gamma \ln \frac{2v_F}{\omega d}}} \tag{28}$$

where q_ω is a solution of equation $\omega = q_\omega v(q_\omega)$, and γ is specified by Eq. (4) and determines to the strength of the long-range Coulomb interaction.

And, finally, let us discuss equations of motion for the single-mode LL, that is spinless or, in other words, spin-polarized LL. This is the simplest case, and derivation of the equations is similar to the spinful case. For the spinless LL we concentrate on the short-range interaction with constant value of K_ρ. Then the equation of motion for the phase at the impurity site reads

$$\partial_t \hat{\Phi}(t) + \int_0^\infty dt_1 W_i Z(t_1) \sin 2\hat{\Phi}(t - t_1) = \frac{1}{2} \int_0^\infty dt_1 F(t - t_1)\hat{P}(t_1). \tag{29}$$

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Equations (22-23) and (29) are non-linear equations for operators, therefore, it is not easy to solve them in general case. As (29) and (22-23) resemble equation of motion of an overdamped pendulum and two coupled pendulums correspondingly, one can expect that when the system is driven by a constant external bias the phase can increase and oscillate, which means presence of both dc and ac current in the system. The simplest case is when the inter-electronic interaction is strong enough ($K_\rho \ll 1$ or γ is of the order or larger then unity). It will be shown that under these conditions fluctuations of the phase field $\hat{\Phi}_\rho$ are relatively small and can be described by Gaussian approximation. We will solve the problem for the case of spinless LL in Gaussian approximation in the next section, and at the end of the section we will discuss the validity of Gaussian approximation. The spinful case is more complicated as fluctuations in the spin channel at $K_\sigma \approx 1$ are not small and are not Gaussian, however, for the considered case of spin-independent interaction, $K_\sigma = 1$, they can be taken into account strictly by means of refermionization method.

III. DYNAMIC REGIME OF CONDUCTION IN THE SPINLESS LUTTINGER LIQUID

A. Gaussian approximation

In this section we will consider the most technically simple case of the single-mode LL with short-range interaction between electrons.

First, we represent the bosonic field operator at the impurity site as a sum of its expectation value and fluctuating part, $\hat{\Phi} = \Phi + \hat{\phi}$, $\Phi = \langle \hat{\Phi} \rangle$. Then we perform thermodynamic averaging of both sides of Eq. (29) and obtain equation for expectation value Φ of the field operator at the impurity site

$$\partial_t \Phi(t) + \int_0^\infty dt_1 W_i Z(t - t_1) \langle \sin 2\hat{\Phi}(t_1) \rangle = \int_0^\infty dt_1 F(t - t_1) V(t_1), \qquad (30)$$

Eq. (30) is not a closed equation for $\Phi(t)$ since it contains an expectation value of $\sin 2\hat{\Phi}(t)$ which depends both on expectation value Φ and on fluctuations $\hat{\phi}$ of the displacement field. Therefore, in order to calculate the expectation value we need to study fluctuations. Subtracting Eq. (30) from Eq. (29) we find an equation of motion for the fluctuating part $\hat{\phi}$

of the displacement field at the impurity site

$$\partial_t \hat{\phi}(t) + \int_0^\infty dt_1 W_i Z(t - t_1)[\sin 2\hat{\Phi}(t_1) - \langle \sin 2\hat{\Phi}(t_1) \rangle] = \int_0^\infty dt_1 F(t - t_1)\delta\hat{P}(t_1), \qquad (31)$$

Equations to be solved somewhat resemble equations for the superconducting phase in the Josephson junctions. However, there is an important difference from Josephson junctions since in the latter case fluctuations of the phase are, typically, small and one can develop equations in powers of small fluctuating phase and consider the Gaussian model for fluctuations. In our case fluctuations are not small, nevertheless, in order to solve the problem we simplify the problem and assume that fluctuations are Gaussian. Strictly speaking, the fluctuations are not Gaussian, and in general case this is just a model assumption. However, we will demonstrate below that this approach can be justified in case of strong inter-electronic repulsion and in the limit of high voltages, where the non-Gaussian part of fluctuations becomes small.

Thus we solve the problem by means of the self-consistent harmonic approximation[1], in which fluctuations are assumed to be Gaussian. In this approximation, we replace

$$\sin 2\hat{\phi} \to 2h\hat{\phi}, \quad h \equiv e^{-2\langle \hat{\phi}^2 \rangle}, \qquad (32)$$

and instead of (30) we obtain more simple equation for the expectation value of the field operator

$$\partial_t \Phi(t) + \int_0^\infty dt_1 W_i Z(t - t_1)h(t_1)\sin 2\Phi(t_1) = \int_0^\infty dt_1 F(t - t_1)V(t_1), \qquad (33)$$

and a linear equation for fluctuations

$$\partial_t \hat{\phi}(t) + 2W_i \int_0^\infty dt_1 Z(t - t_1)h(t_1)\cos 2\Phi(t_1)\hat{\phi}(t_1) = \int_0^\infty dt_1 F(t - t_1)\delta\hat{P}(t_1). \qquad (34)$$

Coefficients of this equation depend both on the mean square fluctuations $\langle \hat{\phi}^2(t) \rangle$ and on the expectation value Φ, so it must be solved self-consistently with Eq. (33).

B. Low applied voltage, $V < V_T$

If the applied dc voltage is small enough, $V < V_T$, Eqs. (33) and (34) have stationary solutions for phase Φ and for mean square fluctuations $\langle \hat{\phi}^2 \rangle$. In this subsection we consider this stationary solution and find V_T. In the stationary case Eq. (33) reads

$$W_i h \sin 2\Phi = V, \qquad (35)$$

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and Fourier transformed Eq. (34) reduces to the simple form

$$-i\omega\hat{\phi}(\omega) + 2W_i h Z(\omega)\cos 2\Phi\hat{\phi}(\omega) = F(\omega)\delta\hat{P}(\omega). \tag{36}$$

This equation can be solved easily. Then taking into account correlation functions given by Eq. (18) we can calculate mean square fluctuations

$$\langle\hat{\phi}^2\rangle = \int \frac{d\omega}{2\pi}\langle\hat{\phi}^2\rangle_\omega.$$

Using Eqs. (36) and (24) the expression under the integral sign can be presented in a convenient form

$$\langle\hat{\phi}^2\rangle = \frac{K_\rho^2}{2}\int_{-\infty}^{\infty} \frac{\omega\coth\frac{\omega}{2T}d\omega}{(\omega^2 + W_c^2)[(1 + K_\rho^2) + (1 - K_\rho^2)\sin(\omega t_L - \alpha_\omega)]}, \tag{37}$$

where

$$\alpha_\omega = \arctan\frac{W_c^2 - \omega^2}{2\omega W_c}, \quad W_c = 2W_i K_\rho h\cos 2\Phi.$$

The result of integration depends on relation between V_T and temperature T. First, we consider the limit of zero temperature. In pure LL this integral would diverge logarithmically both at high and low frequencies. The divergence at the upper limit in the TL formalism must be cut off at frequency Λ of the order of the bandwidth or the Fermi energy. The infrared divergence at low frequencies is a distinctive feature of 1D systems. We find that in the presence of impurity the infrared divergence is cut off at frequency $W_c \propto W_i$. In addition to the logarithmically divergent part, the denominator contains the oscillating factor induced by reflections of fluctuations from contacts. If the length of the quantum wire is large enough, $L \gg v/W_i h$, the main contribution to the integral is determined by frequencies $\omega t_L \gg 1$ and oscillations contribute little to the integral. Then the oscillating factor can be replaced with its average value $K_\rho/2$, and integration yields

$$\langle\hat{\phi}^2\rangle = \frac{K_\rho}{2}\ln\frac{\Lambda}{W_c}. \tag{38}$$

Since W_c depends on $\langle\hat{\phi}^2\rangle$, Eq. (38) determines the self-consistency condition for $\langle\hat{\phi}^2\rangle$. Substituting h from Eq. (32), we find

$$\langle\hat{\phi}^2\rangle = \frac{K_\rho}{2(1 - K_\rho)}\ln\frac{\Lambda}{2W_i K_\rho\cos 2\Phi}. \tag{39}$$

Now using this equation we can calculate maximum value of the left hand side of Eq. (36) which determines the value of the threshold voltage below which the static solutions for mean

phase Φ exist. We find that $W_i h \sin 2\Phi$ reaches its maximum value when $\cos 2\Phi = \sqrt{K_\rho}$, and expression for V_T has a form

$$V_T = 2W_i \left(\frac{2W_i \sqrt{K_\rho^3}}{\Lambda} \right)^{\frac{K_\rho}{1-K_\rho}} \sqrt{1 - K_\rho}. \tag{40}$$

We see that the threshold voltage at low temperatures is determined by the impurity potential renormalized by quantum fluctuations. In case of interelectronic repulsion, $K_\rho < 1$, the mean square fluctuations $\langle \hat{\phi}^2 \rangle$ and, hence, V_T are finite, while in non-interacting system, when $K_\rho = 1$, fluctuations become infinite and V_T is destroyed by quantum fluctuations. Thus we find that the solution for Φ is stationary at $V < V_T$, that is current cannot pass an impurity. This result is a consequence of our approximation in which only Gaussian fluctuations were taken into account. If we took into account fluctuations of solitonic type for which the phase increases by π due to tunneling, we would obtain a small tunneling current at $V < V_T$. Fluctuations of this type yield the well-known power-law I-V curves[1].

Now we consider the case of finite temperatures. The self-consistency equation has solutions which correspond to a finite value of fluctuations only if $T < T_c \sim V_{T,0} \equiv V_T(T = 0, L = \infty)$, so there is a characteristic temperature above which V_T is destroyed by thermal fluctuations and the impurity does not suppress electronic transport.

If the quantum wire is short enough, $L \sim v_\rho / V_{T,0}$, we must not average Eq. (37) over oscillations at $\omega t_L \sim 1$. At these frequencies $\langle \hat{\phi}^2(\omega) \rangle$ in Eq. (37) is proportional to ω^{-1} as before but with a different factor. As a consequence V_T is suppressed in short wires, and impurities do not destroy the linear conduction when $L < L_c \sim v / V_{T,0}$. This happens due to increase of fluctuations at the impurity site because fluctuations are reflected back from the contacts, while the distance to the contacts becomes smaller than the correlation length of the fluctuations.

Thus we have obtained that conductance is greatly suppressed below the threshold field V_T which is determined by impurity potential renormalized by fluctuations. As fluctuations depend on temperature and length of the wire, V_T decreases as temperature increases or the wire becomes shorter. Results of numerical calculations of V_T as a function of temperature and wire length for $K_\rho = 0.3$ are presented in Fig. 1. Results for other values of K_ρ are qualitatively similar. One can see that there are critical temperature and the inverse wire length above which there are no solutions with non-zero V_T.
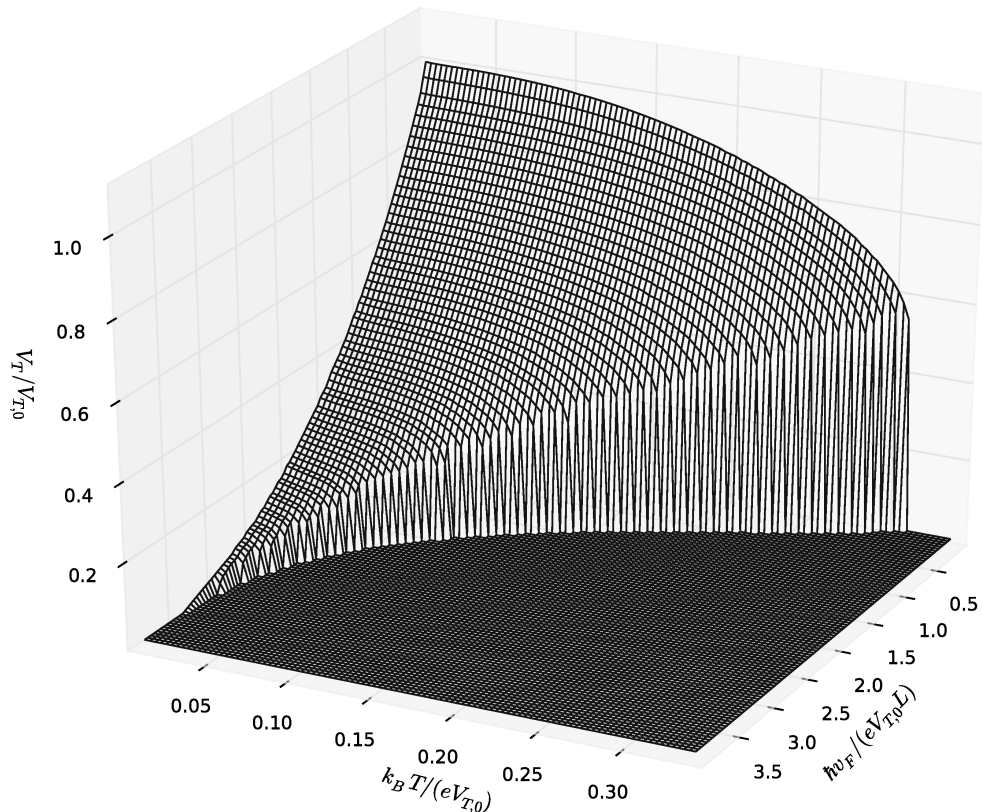
FIG. 1. Results of numerical calculations of the normalized threshold field as function of temperature and wire length

In Fig. 2 we present a "phase diagram" which shows characteristic temperatures and wire lengths limiting the region $V_T > 0$ where conductivity is suppressed by impurity.

C. Dynamical regime near the threshold voltage

Now we consider the behavior of the system at voltages slightly above the threshold value at low temperatures. As equation of motion (33) resembles equation of motion of an overdamped pendulum, near the threshold the phase must increase very non-uniformly, since under such conditions the pendulum would slowly move up and rapidly fall down. So we expect that when $\delta V \equiv V - V_T \ll V_T$, the phase increases very slowly if $\sin 2\Phi$ is close to unity, and rapidly passes the values of Φ for which $V - V_T \sin 2\hat{\Phi}$ is large in comparison with δV. Then as the phase increases slowly, fluctuations can be described by means of
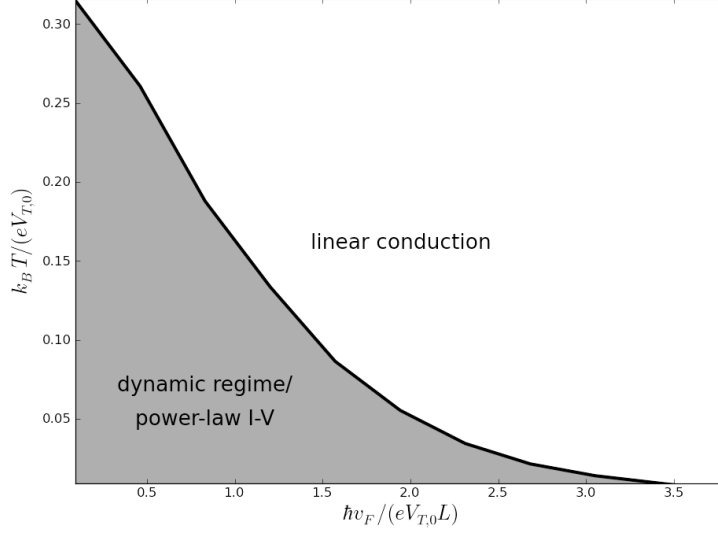
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FIG. 2. Region of temperatures and wire lengths in which the impurity strongly affects the electronic transport

quasi-static adiabatic approach and we can use Eq. (39). The latter is applicable provided the values of Φ satisfy condition $\cos 2\Phi > 0$. Note that the maximum value of $W_i h \sin 2\Phi$ is determined by $\cos 2\Phi_{0m} = \sqrt{K_\rho}$, that is $\cos 2\Phi$ is positive in the vicinity of the maximum. Then in the region around the maximum where the phase spends the most part of the period we can, indeed, use the adiabatic approach. In this region we can develop $V_T \sin 2\Phi$ near its maximum in series with respect to $\delta\Phi = 2(\Phi_{0m} - \Phi)$ and obtain equation of motion for the phase in this region

$$\partial_t \delta\Phi + V_T \left(1 - \frac{\delta\Phi^2}{1 - K_\rho}\right) = V. \tag{41}$$

Its solution around the maximum has the form

$$\delta\Phi = \sqrt{\frac{(V - V_T)(1 - K_\rho)}{V_T}} \tan \sqrt{\frac{(V - V_T)V_T}{1 - K_\rho}} t. \tag{42}$$

We see that $\delta\Phi$ is small during large time $t_0 \sim \pi\sqrt{\frac{1 - K_\rho}{(V - V_T)V_T}}$ and then $\delta\Phi$ increases rapidly during short time, so that in the first approximation in $\sqrt{(V - V_T)/V_T}$ the period of oscillations is determined by time t_0. Hence, with the same accuracy we can calculate dc current

$$\bar{I} = 2G_0 \sqrt{\frac{(V - V_T)V_T}{1 - K_\rho}}. \tag{43}$$

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We can also conclude that time dependence of the current consists of narrow peaks of height $\sim G_0 V_T$ and width $\sim 1/V_T$ separated by long time t_0 during which the current is small.

D.   I-V curves and noise spectrum at high voltages

As it was noted already, it is difficult to obtain I-V curves at low voltages accurately because of time dependence of the mean square value of fluctuations. The problem is simplified at high voltages, $V \gg V_T$, when the mean square value $\langle \hat{\phi}^2 \rangle$ becomes nearly constant with small oscillating component. In this case Eqs. (33-34) can be solved perturbatively assuming that the oscillating parts of both mean square fluctuations $\langle \hat{\phi}^2 \rangle$ and of the mean phase Φ are small.

In this subsection we consider the limit of relatively long conducting channel, $V_T t_L \gg 1$, but not too long, so that the wire is short in comparison with the damping length related to relaxation due to coupling to phonons etc, $L \ll l_\nu$. In this case we have to use the exact form of $Z(t)$ in equation for the expectation value Eqs. (33) but can keep only the first delta-function in kernel $Z(t)$ in equation for fluctuations Eq. (34). In time representation this means that we take into account current pulses reflected from the contacts but we ignore correlations between fluctuations shifted by time $n t_L$ necessary for the excitation to return to the impurity after multiple reflections from the contacts. Then Eq. (34) acquires simple form and can be solved easily

$$\partial_t \hat{\phi} + W(t)\hat{\phi}(t) = \hat{f}(t), \quad W(t) = 2K_\rho W_i h(t) \cos 2\Phi(t), \quad \hat{f}(t) \equiv \int_0^\infty dt_1 F(t - t_1)\delta\hat{P}(t_1).$$
(44)

Solution of Eq. (44) has the form

$$\hat{\phi} = \int_{-\infty}^t dt_1 \hat{f}(t_1) e^{-\int_{t_1}^t W(t_2)dt_2}.$$
(45)

Using Eq. (45) we can calculate mean square fluctuations $\langle \hat{\phi}^2 \rangle$. As we consider the long channel we average, again, over oscillatory factor in $F(t)$ and find

$$\langle \hat{\phi}^2 \rangle = \frac{K_\rho}{4} \int_{-\infty}^t dt_1 \int_{-\infty}^t dt_3 \int d\omega \omega \coth \frac{\omega}{2T} e^{-\int_{t_1}^t W(t_2)dt_2 - \int_{t_3}^t W(t_2)dt_2 - i\omega(t_1 - t_3)}.$$
(46)

To solve this equation we need to calculate, first, $W(t)$ which is determined by fluctuations. In order to do this we solve Eqs. (33) and (46) for fluctuations seeking for $\langle \hat{\phi}^2 \rangle$ in the form $\langle \hat{\phi}^2 \rangle = \langle \hat{\phi}^2 \rangle_t + c \cos \omega_0 t + s \sin \omega_0 t$, where $\omega_0 \equiv 2\pi \bar{I}$, and $\langle \cdots \rangle_t$ denotes averaging in time.

We assume also that $c, s \ll 1$. Substituting this form into Eq. (46) and keeping only leading terms we obtain in the limit of low temperatures

$$\langle \hat{\phi}^2 \rangle = \frac{K_\rho}{4} \int_{-\infty}^{\infty} \frac{|\omega| d\omega}{\omega^2 + b^2} \left[1 - \frac{2W_0 \left[b(\omega_0^2 + \omega^2) \cos \omega_0 t + \omega_0 (\omega_0^2 - \omega^2) \sin \omega_0 t \right]}{(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2} \right], \qquad (47)$$

where $W_0 = 2W_i K_\rho e^{-2\langle \hat{\phi}^2 \rangle_t}$, $b = \langle W(t) \rangle_t = |c|W_0$. Performing integration in Eq. (47) we obtain

$$\langle \hat{\phi}^2 \rangle = \frac{K_\rho}{2} \left[\ln \frac{\Lambda}{b} - \frac{\pi W_0}{\omega_0} \cos \omega_0 t - \frac{2W_0}{\omega_0} \ln \frac{\omega_0}{b} \sin \omega_0 t \right]. \qquad (48)$$

Thus we have found that the main logarithmic contribution to $\langle \hat{\phi}^2 \rangle_t$ is determined by relation similar to Eq. (38) valid in case of small voltages, but with different infrared cut-off frequency b which is much smaller than W_c in Eq. (38). From the self-consistency condition we find

$$c = -\frac{\pi K_\rho W_0}{2\omega_0}, \quad s = -\frac{K_\rho W_0}{\omega_0} \ln \frac{2\omega_0^2}{\pi W_0^2}, \quad W_0 = V_T \frac{K_\rho}{\sqrt{1 - K_\rho}} \left(\frac{\pi \sqrt{K_\rho^3} V_T}{2\sqrt{1 - K_\rho} V} \right)^{\frac{K_\rho}{1 - 2K_\rho}} \ll V_T. \qquad (49)$$

Here we have expressed W_0 from Eq. (40) in terms of V_T at zero temperature in the limit of the long wire.

We see that at high voltages the solution with finite amplitude of the oscillations exists only at $K_\rho < 1/2$, i.e., when inter-electronic interaction is strong enough. The result differs from that for the regime of small voltages when fluctuations do not destroy the dynamic regime at any repulsion strength, $K_\rho < 1$. The result also differs from the case of dynamic regime of conduction induced by Friedel oscillations at sharp contacts to quantum wire[33], where the critical value at high voltages is $K_\rho = 1/3$.

So far we considered in this subsection long channels at the zero temperature limit. As fluctuations at high voltages become stronger, they decrease the "critical" temperature and increase the "critical" length in comparison with the case of small voltages considered in subsection III B. Therefore, in the limit of high voltage and low temperatures we obtain $T_c \sim W_0$ and $L_c \sim vV/W_0^2$.

Now, when we have found mean square fluctuations (48), we can solve Eq. (33) perturbatively in the limit of high voltages, $V \gg V_T$, and calculate current using Eq. (8). The total current passing the impurity consists of dc part, $\bar{I} = VG_0 + I_{nl}$, where I_{nl} is non-linear correction to Ohm's law, and of ac part, $I_{ac} \sin \omega_0 t$, which oscillates with frequency

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$\omega_0 = 2\pi\bar{I}/e \approx eV/\hbar$ (in dimensional units)

$$I_{ac} = G_0 W_0 \sqrt{\frac{(1 + K_\rho^2) + (1 - K_\rho^2)\cos\omega_0 t_L}{(1 + K_\rho^2) - (1 - K_\rho^2)\cos\omega_0 t_L}}, \tag{50}$$

$$I_{nl} = -\frac{2G_0 W_0^2}{V}\left[\ln\frac{2V^2}{\pi W_0^2} + \frac{1}{(1 + K_\rho^2) - (1 - K_\rho^2)\cos\omega_0 t_L}\right]. \tag{51}$$

The oscillating factors in these expressions are due to reflections of current pulses generated at the impurity from the contacts. Non-linear correction to Ohm's law is shown in Fig. 3.
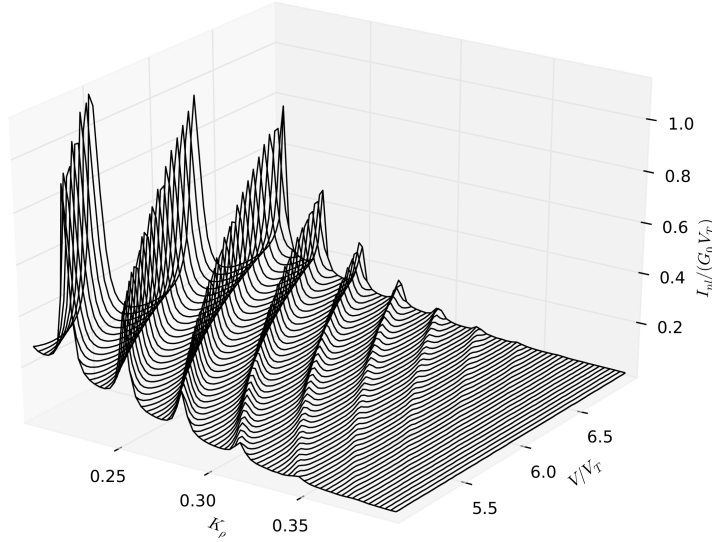

FIG. 3. Dependence of non-linear part of the current, I_{nl}, on voltage and interaction parameter

The ac current depends on the coordinate along the wire, and its value at the contacts to the wire is different from that at the impurity site given by Eq. (50). The amplitude of the ac current at the contacts reads

$$I_{ac-contact} = \frac{\sqrt{2}G_0 W_0}{\sqrt{1 + K_\rho^2 - (1 - K_\rho^2)\cos\omega_0 t_L}}$$

Now we calculate the noise spectrum around the oscillation frequency

$$\langle\delta\hat{I}(\omega)\delta\hat{I}(\omega')\rangle = \int dt \int dt' \langle\delta\hat{I}(t)\delta\hat{I}(t')\rangle e^{i\omega t}\int\frac{d\omega'}{2\pi}e^{i\omega't'}. \tag{52}$$

Then we express $\delta\hat{I}(t) = \partial_t\hat{\phi}/\pi$ from Eq. (31) and find that

$$\langle\delta\hat{I}(t)\delta\hat{I}(t')\rangle = (W_i/\pi)^2[\langle\sin 2\hat{\Phi}(t)\sin 2\hat{\Phi}(t')\rangle - \langle\sin 2\hat{\Phi}\rangle^2]. \tag{53}$$

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We calculate correlation function in the right-hand side of Eq. (53) assuming that fluctuations are Gaussian

$$\langle \sin 2\hat{\phi}(t) \sin 2\hat{\phi}(t') \rangle = \frac{1}{2} \cos 2(\Phi(t) - \Phi(t')) e^{2\langle [\hat{\phi}(t) - \hat{\phi}(t')]^2 \rangle}. \tag{54}$$

Correlation function of the phases can be found from the first term in Eq. (47) that determines the leading contribution

$$\langle \hat{\phi}(t) \hat{\phi}(t') \rangle = -\frac{K_\rho}{4} \left[e^{-b\tau} \text{Ei}(b\tau) + e^{b\tau} \text{Ei}(-b\tau) \right], \quad \tau \equiv t - t'. \tag{55}$$

Using Eqs. (52-55) and calculating the integrals we find two maxima of the noise spectrum around frequencies $\omega = \pm\omega_0$

$$\langle \delta\hat{I}(\omega)\delta\hat{I}(\omega') \rangle \approx G_0^2 V_T^{2(1-K_\rho)} \frac{\pi\Gamma(1 - 2K_\rho)\sin\pi K_\rho}{2(1 - K_\rho)^{1-K_\rho} K_\rho^{3K_\rho} ||\omega| - \omega_0|^{1-2K_\rho}} \delta(\omega + \omega'). \tag{56}$$

Note that the maxima are present under the same condition $K_\rho < 1/2$ for which the solution with finite amplitude of the oscillations at high voltages was found. According to Eq. (56) the integral noise power is of the order of $\sim G_0^2 W_i^2$ which is much larger than the ac signal power $\sim I_{ac}^2$ at frequency ω_0.

In case of long-range Coulomb interaction the correlation function can be found similarly to derivation of Eq. 55, it has the form

$$\langle \hat{\phi}(t) \hat{\phi}(t') \rangle = \frac{1}{4\gamma} \ln \ln \frac{\Lambda\sqrt{\ln^3 \frac{\Lambda}{V}}}{W_i^2 |t - t'|}.$$

Then instead of Eq. 56 we find at $\omega \gg v_F/L$ when $Z(\omega)$ is described by Eq. (28)

$$\langle \delta\hat{I}(\omega)\delta\hat{I}(\omega') \rangle \sim \frac{W_i^2}{8\gamma||\omega| - \omega_0| \ln \frac{2v_F}{\omega d}} \left(\ln \frac{|\omega|||\omega| - \omega_0|}{W_i^2} \right)^{\frac{1}{4\pi\gamma}} \delta(\omega + \omega').$$

The noise spectrum decreases from its maximum value approximately as $1/\omega$. This is somewhat similar to the case of very strong short-range ineraction ($K_\rho \to 0$) in Eq. 56.

E. Validity of Gaussian approximation

Now we discuss conditions under which the Gaussian model that we have used to describe fluctuations can be justified quantitatively. Note that fluctuations of the displacement field $\hat{\phi}$ in pure 1D system are Gaussian because the TL Hamiltonian is quadratic, and mean square

fluctuations are infinite, $\langle \hat{\phi}^2 \rangle = \infty$. Impurity makes fluctuations at the impurity site finite (confer Eq. 38), but fluctuations become non-Gaussian because of the cosine impurity term in the Hamiltonian. As the current passes the impurity, the impurity term oscillates, and frequency of the oscillations increases with voltage increasing. This results in a decrease of the time-averaged impurity potential making the impact of the impurity effectively smaller. Therefore one should expect that relative contribution of the non-Gaussian part to fluctuations must decrease in comparison with the Gaussian part. Then at voltages $V \gg V_T$ we can try to calculate non-Gaussian contribution to fluctuations perturbatively.

We select two contributions of the fluctuating part of the phase, $\hat{\phi} = \hat{\phi}_G + \hat{\phi}_1$, where the first term is the Gaussian contribution which satisfies simplified Eq. (34), while $\hat{\phi}$ satisfies full Eq. (31). Considering non-Gaussian part $\hat{\phi}_1$ as a small correction we linearize Eq. (31) and obtain equation for $\hat{\phi}_1$. We consider, again, zero temperature and long conducting channel, $V_T t_L \gg 1$, when $Z(t) \approx K_\rho \delta(t)$. Then equation for non-Gaussian corrections reads

$$-i\omega \hat{\phi}_1 + W(t)\hat{\phi}_1(t) = W_i K_\rho \times$$
$$\left\{ \cos 2\Phi(t)[2h(t)\hat{\phi}_G(t) - \sin 2\hat{\phi}_G(t)] + \sin 2\Phi(t)[\langle \cos 2\hat{\phi}_G(t) \rangle - \cos 2\hat{\phi}_G(t)] \right\}. \quad (57)$$

Next, we derive equation of motion for the third cumulant C_3. In the first approximation, $C_3(t) = \langle \hat{\phi}_1(t)\hat{\phi}_G(0)^2 \rangle$. We multiply Eq. (57) by $\phi_G^2(t = 0)$, and make thermodynamic averaging using for the right-hand side the rules valid for Gaussian fluctuations. Then we obtain

$$\partial_t C_3(t) + W(t)C_3(t) = 4W_i K_\rho h(t) \sin 2\Phi(t) \langle \{\phi_G(t)\phi_G(0)\} \rangle^2.$$

Solution of this equation has the form

$$C_3(t) = -\int_{-\infty}^t dt_1 e^{-\int_{t_1}^t W(t_2)dt_2} 4W_i K_\rho h(t_1) \sin 2\Phi(t_1) \langle \{\phi_G(t_1)\phi_G(0)\} \rangle^2$$

Calculating the integral, using Eq. (55), and keeping the leading terms we find

$$C_3(0) = a_3 K_\rho \left[1 - K_\rho \ln \frac{4V^2}{\pi W_0^2} \right], \quad a_3 = \frac{1}{4\pi} \int_0^\infty dx e^{-x} \left[e^{-x}\text{Ei}(x) + e^x \text{Ei}(-x) \right]^2 \approx 0.3490658. \quad (58)$$

Similarly, from Eq. (57) we can calculate the fourth cumulant $C_4 = \langle \hat{\phi}_1 \hat{\phi}_G(0)^3 \rangle$. The result for the cumulant at coinciding times is

$$C_4(0) = a_4 K_\rho^3, \quad a_4 = -\frac{1}{16} \int_{-\infty}^0 \left[e^{-x}\text{Ei}(x) + e^x \text{Ei}(-x) \right]^3 e^x dx \approx 2.7224372. \quad (59)$$

Now we can compare non-Gaussian contributions (58, 59) with related Gaussian contributions. The third cumulant contains two terms. The first one does not increase with voltage, while the second term increases with voltage but more slowly than $\langle \hat{\phi}^2 \rangle$ given by Eq. (48) and contains additional small factor K_ρ. Non-Gaussian contribution (59) contains both small factor K_ρ and does not increase with voltage. Thus we see that non-Gaussian contributions (58, 59) are relatively small. For example, the non-Gaussian contribution to $\langle \hat{\phi}^4 \rangle$ given by Eq. (59) is small in comparison with Gaussian contribution, $3 \langle \hat{\phi}^2 \rangle^2 \sim K_\rho^2 \ln^2 \left[\left(\frac{\Lambda}{V_T} \right) \left(\frac{V}{V_T} \right)^{\frac{1}{1-2K_\rho}} \right]$, which increases with voltage.

IV. DYNAMIC REGIME OF CONDUCTION IN THE SPINFUL LUTTINGER LIQUID

A. Refermionization in the spin channel

In Sec. III we treated fluctuations in the Gaussian approximation which can be justified only in the limits of strong interaction at high voltages. In the spin channel, fluctuations at the impurity site are always strong and non-Gaussian. Therefore, though the Gaussian approximation can be used as a model one, it is not justified. However, if interaction is spin-rotation invariant ($K_\sigma = 1$) and the impurity is situated in the middle of the wire we can make use of the symmetry of the system and solve the problem strictly by means of refermionization method. This method consists in introducing new fermionic variables for spin channel. Equations of motion for these variables are linear, and, hence, soluble. Refermionization was used successfully to treat charge fluctuations in the spinless case for the specific value of interaction parameter $K_\rho = 1/2$ (Ref. 24 and 35) and to describe spin fluctuations in the spinful case for $K_\sigma = 1$ (Ref. 36 and 39). Following the approach of Ref.24 we introduce new phase fields by relation that differs from the similar one in Ref. 24 by a constant factor

$$\hat{\phi}_\pm(x) = \frac{1}{\sqrt{2}} \left[\hat{\Phi}_\sigma(x) + \hat{\Theta}_\sigma(x) \right] \pm \frac{1}{\sqrt{2}} \left[\hat{\Phi}_\sigma(-x) - \hat{\Theta}_\sigma(-x) \right]. \tag{60}$$

New fields are completely decoupled and the impurity term couples to the field $\hat{\phi}_+$ only. Then following Matveev[36] and Egger and Grabert[24] we introduce new fermion variables $\sqrt{\frac{1}{2\pi a}} e^{i\hat{\phi}_+} = \hat{g}\hat{\psi}, \quad \hat{g} = \hat{c} + \hat{c}^\dagger$, where $\hat{g}/\sqrt{2}$ is an auxiliary Majorana fermion operator. Then

we derive equations of motion for Heisenberg operators $\hat{\psi}$ and find that they depend on $x - v_F t$. Equations of motion for operators $\hat{\psi}_{1,2}(t) = \hat{\psi}(x = \mp 0, t)$ at the impurity site and for $\hat{g}$ have the form

$$v_F(\hat{\psi}_2 - \hat{\psi}_1) = i\hat{g}f, \quad v_F(\hat{\psi}_2^\dagger - \hat{\psi}_1^\dagger) = -i\hat{g}f \tag{61}$$

$$\partial_t \hat{g} = i[f(\hat{\psi}_1 + \hat{\psi}_2) - f(\hat{\psi}_1^\dagger + \hat{\psi}_2^\dagger)], \tag{62}$$

where $f(t) = \sqrt{2\pi a} W \cos\sqrt{2}\hat{\Phi}_\rho$.

Density perturbations of new fermions are related in a standard way to the gradient of the displacement field

$$\hat{\psi}_x^+ \hat{\psi}_x - \langle \hat{\psi}_x^+ \hat{\psi}_x \rangle_0 = \frac{1}{2\pi} \partial_x \hat{\phi}_+(x). \tag{63}$$

We will consider the limit of strong interaction between electrons when fluctuations in the charge channel are small. We represent the field operator at the impurity site again as a sum of its expectation value and fluctuating part, $\hat{\Phi}_\rho = \Phi_\rho + \hat{\phi}_\rho$, $\Phi_\rho = \langle \hat{\Phi}_\rho \rangle$ and take into account fluctuations $\hat{\phi}_\rho$ only up to the first approximation. Then commutators of f at different times are small and we can ignore time-ordering and solve equation (62) for $\hat{g}(t)$ as if $f(t)$ is a c-number

$$\hat{g} = 2i \int^t dt_1 [f(t_1)\hat{\psi}_1(t_1) - f(t_1)\hat{\psi}_1^\dagger(t_1)] \exp\left[-\frac{2}{v_F} \int_{t_1}^t f(t_2)^2 dt_2\right]. \tag{64}$$

Now by means of Eq. (61) we can express operators $\hat{\psi}(x = 0) \equiv (\hat{\psi}_1 + \hat{\psi}_2)/2$. Then using

$$e^{i\hat{\phi}_+} = \sqrt{2\pi a}\,\hat{g}\hat{\psi} = \frac{1}{2}\sqrt{2\pi a}\,\hat{g}(\hat{\psi}_1 + \hat{\psi}_2), \quad e^{-i\hat{\phi}_+} = -\sqrt{2\pi a}\,\hat{g}\hat{\psi}^\dagger = -\frac{1}{2}\sqrt{2\pi a}\,\hat{g}(\hat{\psi}_1^\dagger + \hat{\psi}_2^\dagger) \tag{65}$$

we can find $\cos\sqrt{2}\hat{\Phi}_\sigma$ which we insert into the equation of motion for the charge phase (22)

$$\cos\sqrt{2}\hat{\Phi}_\sigma = \sqrt{\frac{\pi a}{2}}\,\hat{g}(\hat{\psi} - \hat{\psi}_1^\dagger) \tag{66}$$

where we made use of the anticommutator relation $\{\hat{g}(t), \hat{\psi}_1^\dagger(t)\} = \frac{if}{v_F}$.

We obtain the following expression for $\cos\sqrt{2}\hat{\Phi}_\sigma$:

$$\cos\sqrt{2}\hat{\Phi}_\sigma(t) = 2i\pi a W \int_{-\infty}^t dt_1 \cos\sqrt{2}\hat{\Phi}_\rho(t_1) \tag{67}$$

$$\times \left\{ [\hat{\psi}_1(t_1) - \hat{\psi}_1^\dagger(t_1)]\hat{\psi}_1(t) + \hat{\psi}_1^\dagger(t)[\hat{\psi}_1(t_1) - \hat{\psi}_1^\dagger(t_1)] \right\} \exp\left[-\frac{2}{v_F} \int_{t_1}^t f(t_2)^2 dt_2\right].$$

We insert (67) into the equation of motion for the charge phase (22). We consider the limit of small fluctuations. In this case averaging over charge and spin variables can be performed

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separately since the fluctuations in spin and charge sectors are independent. Expectation values of fermionic densities in (67) can be associated with distribution function of new fermions by relations

$$\langle \hat{\psi}_1^\dagger(t_1), \hat{\psi}_1(t_2) \rangle = \int \frac{d\varepsilon}{2\pi v_F} n(\varepsilon, t) e^{i\varepsilon(t_1 - t_2)}, \quad \langle \hat{\psi}_1(t_1), \hat{\psi}_1^\dagger(t_2) \rangle = \int \frac{d\varepsilon}{2\pi v_F} [1 - n(\varepsilon, t)] e^{-i\varepsilon(t_1 - t_2)},$$
(68)

where $t = (t_1 + t_2)/2$, and other pairings are equal to zero. This is true because operators with subscript 1 are related to the incident spin excitations. These excitations are not affected by the impurity because the coefficient of reflection from the contact $r = \frac{1 - K_\sigma}{1 + K_\sigma}$ is equal to zero for $K_\sigma = 1$. Note that this case differs from the problem for charge channel considered in Ref. 24. Charge excitations incident on the impurity contain fraction of excitations which reflected from the contact and, therefore, interacted with the impurity.

Now we need to determine distribution function $n(\varepsilon, t)$. To do this we, first, subtract boundary conditions (20) at $x = -L/2$ and $x = -L/2$ for spin sector and obtain

$$v_F \partial_x \hat{\phi}_+ \left(-\frac{L}{2}, t \right) = \hat{P}_\sigma.$$
(69)

Then we express the derivative $\partial_x \hat{\phi}_+$ in terms of fermion density from (63) and take the expectation value. In this way we find the condition for the fermion density expressed in terms of the distribution function

$$\int \frac{d\varepsilon}{2\pi} [n(\varepsilon, t) - n_F(\varepsilon)] = V_\sigma(t).$$
(70)

Next we multiply equations (69) taken at different times t_1 and t_2 and calculate the expectation value. Reducing products of four fermions to sum of products of pairs in a standard way and using correlation functions (18) we arrive at the kinetic equation for the distribution function

$$\int n(\varepsilon - \omega, t)[1 - n(\varepsilon, t)] d\varepsilon = \frac{\omega}{2} \left(1 + \coth \frac{\omega}{2T} \right).$$
(71)

Solution of equations (70-71) is

$$n(\varepsilon, t) = \frac{1}{1 + e^{\frac{\varepsilon - V_\sigma(t)}{T}}}.$$
(72)

Thus distribution function of new fermions has a form of the equilibrium function with the chemical potential equal to spin bias. Note that the distribution function has such a form because at $K_\sigma = 1$ there are no reflections of excitations from the contacts. In case of

spinful electrons with $K_\rho = 1/2$ we would obtain kinetic equation different from (71) which does not have solution in the form of the equilibrium distribution because particles, incident on the impurity, contain a fraction that passed the impurity and then reflected from the contact. Reflection coefficient in this case is equal to $r = \frac{1-K_\rho}{1+K_\rho} = \frac{1}{3}$. Therefore, the form of the distribution function of fermions assumed in Ref. 24 needs some justification.

Using equilibrium distribution function (72) in (68) we insert (67) into (22). Then we perform integration over energies and find closed equation for the charge phase

$$\partial_t \hat{\Phi}_\rho + \frac{w}{\sqrt{2}} Z \otimes \sin \sqrt{2}\hat{\Phi}_\rho(t) \int_0^\infty dt_1 \frac{T \cos \sqrt{2}\hat{\Phi}_\rho(t-t_1)}{\sinh \pi T t_1} e^{-2w \int_{t-t_1}^t \cos^2 \sqrt{2}\hat{\Phi}_\rho(t_2)dt_2}$$
$$\times \cos V_\sigma \left(t - \frac{t_1}{2}\right) t_1 = F \otimes \hat{P}_\rho, \tag{73}$$

where $w = 2\pi a W^2/v_F$ is the characteristic potential which results from the impurity potential renormalized by spin fluctuations. This expression is strict in the limit of strong interaction, and now we will discuss the conditions of validity of our approach that assumes smallness of the fluctuations at the impurity site.

To estimate fluctuations we will simplify equation (73) taking into account logarithmic divergence of the integral at $t_1 = 0$. Then with the logarithmic accuracy we perform integration neglecting t_1 dependence of the regular part of the integrand and using the standard ultraviolet cut-off of the integration at $t_1 \sim 1/\Lambda$. We obtain

$$\partial_t \hat{\Phi}_\rho + V_0 Z \otimes \sin 2\sqrt{2}\hat{\Phi}_\rho = F \otimes \hat{P}_\rho, \quad V_0 = \frac{w}{2\pi\sqrt{2}} \ln \frac{\Lambda}{w}, \tag{74}$$

This equation is similar to Eq.(29) for a single-mode LL and can be made identical to (29) by changing notations. Therefore, for the case of short-range interaction we can use the results of Sec. III. Then using Eq. (38) we find that in the limit of low voltages fluctuations are small provided $K_\rho \ln \frac{\Lambda}{w} \ll 1$, while from Eqs. (48-49) we find that in the limit of large voltages fluctuations are small under condition $K_\rho \ln \frac{\Lambda \omega_0}{w^2} \ll 1$.

Let us estimate now the strength of fluctuations in case of long-range interactions. We solve Eq. 74 in linear approximation in fluctuating part of the displacement field $\hat{\phi}_\rho = \hat{\Phi}_\rho - \Phi_\rho$, $\Phi_\rho = \langle \hat{\Phi}_\rho \rangle$ and find

$$\hat{\phi}_\rho = \frac{F(\omega)\delta\hat{P}_\rho}{-i\omega + C} \quad C = 2\sqrt{2}V_0 \langle Z \otimes \cos 2\sqrt{2}\Phi_\rho \rangle_t,$$

where $\langle\rangle_t$ means time-averaging. To calculate constant C we must solve Eq. 74 for expectation value Φ_ρ. Here we will assume that temperature T is low enough, $T \ll V_0$, and limit our estimation by the cases of small and large voltages.

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According to study of the dynamics in Sec. III, we can conclude that at small voltages below the threshold Φ_ρ is constant. Then we obtain with logarithmic accuracy again

$$\langle \delta\hat{\Phi}_\rho^2 \rangle \approx \frac{1}{4\gamma} \ln\ln\left(\frac{v_F\sqrt{\gamma}}{dV_0}\right).$$

In the limit of large voltages the phase increases linearly $2\sqrt{2}\Phi \approx \omega_0 t$ with $\omega_0 = 2\pi f = 2\pi\bar{I} \approx 2V$. Estimating $F(\omega)$ and $Z(\omega)$ we find

$$C \approx \frac{V_T}{\omega_0\sqrt{2\gamma\ln^3\frac{2}{q_\omega d}}}, \quad q_\omega \approx \frac{\omega}{v_F\sqrt{2a\ln\frac{2v_F}{\omega d}}}.$$

Then using Eq. 18 we obtain with logarithmic accuracy

$$\langle \delta\hat{\Phi}_\rho^2 \rangle \approx \frac{1}{4\gamma} \ln\ln\left(\frac{v_F\omega_0\sqrt{\gamma}}{dV_0^2}\right).$$

From these estimates we conclude that in case of long-range Coulomb interaction fluctuations of the displacement field at the impurity site in the charge channel are not large even at moderate values of parameter γ of the order of unity or larger, which correspond to typical values of Fermi velocity (confer Eq. 4), and under this condition equations of motion can be treated in quasiclassical limit. Though fluctuations increase with voltage increasing the dependence is very weak since ω_0 is inside the double logarithm.

B.   I-V curves and current oscillations

To calculate current in the system we must solve Eq. 73 in which we neglect fluctuations and substitute $\hat{\Phi}_\rho$ by its expectation value Φ_ρ. But it is not simple to solve it analytically. Therefore, we restrict our study by limiting cases.

The simplest case is the regime of current bias. We consider this regime and assume that V_σ is time-independent.

$$V(t) = \frac{\omega_0}{2} + \frac{w}{\sqrt{2}}\int_0^\infty d\tilde{t}\, Y(\tilde{t})\sin\frac{\omega_0(t-\tilde{t})}{2}\int_0^\infty dt_1 \cos\frac{\omega_0(t-\tilde{t}-t_1)}{2}\cos V_\sigma t_1$$

$$\times\frac{T}{\sinh \pi T t_1}\exp\left\{-wt_1 - \frac{2w}{\omega_0}\sin\frac{\omega_0 t_1}{2}\cos\left[\frac{\omega_0 t_1}{2} + \omega_0(t-\tilde{t})\right]\right\}, \tag{75}$$

where $Y(\omega) = Z(\omega)/F(\omega)$.

If we perform time averaging of (75) we find the static I-V curves.

$$V_{dc} = \frac{\omega_0}{2} \tag{76}$$
$$+ \frac{w}{4}\int_0^\infty dt_1 \frac{T\cos V_\sigma t_1}{\sinh \pi T t_1}\left[\sin\omega_0 t_1\, I_1\left(\frac{2w}{\omega_0}\sin\frac{\omega_0 t_1}{2}\right) + \sin\frac{\omega_0 t_1}{2}\, I_0\left(\frac{2w}{\omega_0}\sin\frac{\omega_0 t_1}{2}\right)\right]e^{-wt_1}.$$

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It is worth noting that the result is the same for both short-range and long-range interaction, which is not very strange since we consider here the limit of strong interaction, and fluctuations in the charge channel are neglected.

I-V-curves can be presented in simple analytical form in limiting cases. Consider $T \ll w$. In the limit of small current, $\omega_0 \ll w$, the second term dominates over the first linear term and current is proportional to $\sqrt{\frac{\omega_0}{w}}$. This corresponds to the result $I \propto \sqrt{V}$ expected for the case $K_\sigma = 1$ and $K_\rho \to 0$.

The results for the opposite limit of large currents, $\omega_0 \gg w$, are similar in cases of both voltage and current bias. At low temperatures, $T \ll w$, we obtain

$$V_{dc} \approx \frac{\omega_0}{2} + \frac{w}{8} \left[1 - \frac{1}{\pi} \arctan \frac{4\omega_0 w}{\omega_0^2 - V_\sigma^2} \right]. \tag{77}$$

The asymptotic I-V curve is parallel to the Ohm's law corresponding to conductance quantum $2G_0 = e^2/(\pi\hbar)$ with the excess voltage $V_{exc} = \frac{w}{8}$. The excess voltage does not depend on temperature, at temperatures $T > w$ its asymptotic behavior is

$$V_{exc} = \frac{w}{8} \tanh \frac{\omega_0}{4T}.$$

The general view of the I-V curves is presented in Figs. 4.

Time dependence of voltage (75) can be characterized by amplitudes of harmonics $n > 0$ which for $V_\sigma = 0$ are

$$V_n = \frac{\omega_0}{2} \delta_{0n} + \frac{w}{4} Y(n\omega_0) \int_0^\infty dt_1 \frac{T \cos V_\sigma t_1}{\sinh \pi T t_1} \left\{ \sin \frac{\omega_0 t_1}{2} I_{|n|}(z) \right. \tag{78}$$
$$\left. - \frac{1}{2} \sin \omega_0 t_1 \left[I_{|n+1|}(z) + I_{|n-1|}(z) \right] + \frac{i}{2} \cos \omega_0 t_1 \left[I_{|n+1|}(z) - I_{|n-1|}(z) \right] \right\} e^{-wt_1 - in\frac{\omega_0 t_1}{2}},$$

with

$$z = -\frac{2w}{\omega_0} \sin \frac{\omega_0 t_1}{2}.$$

Here the difference between short-range and long-range interactions is present since $Y(\omega) = Z(\omega)/F(\omega)$ in these cases is different.

At small currents, $\omega_0 \ll w$, amplitude of harmonics decays slowly, approximately as $1/\sqrt{n}$. In the limit of large currents, $\omega_0 \gg w$, harmonics decay as power law, and with logarithmic accuracy we obtain for $n > 0$

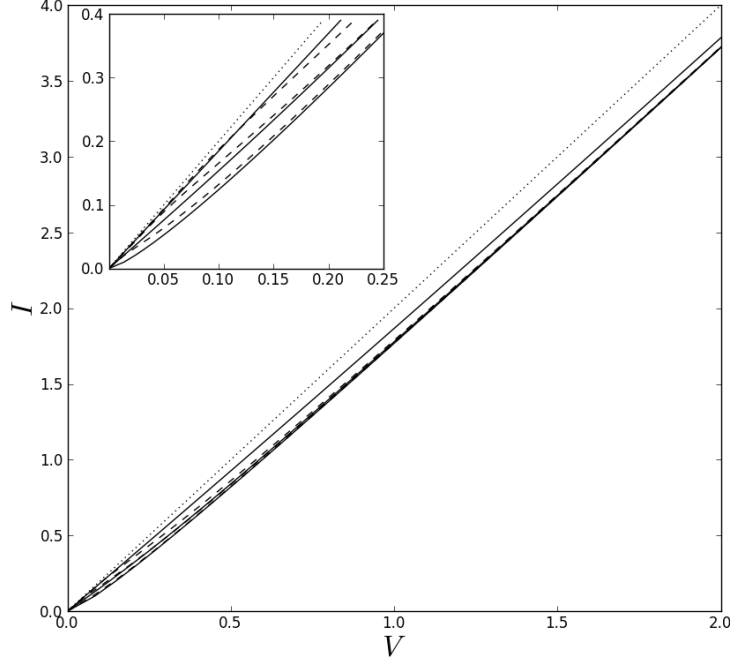$$V_n \approx \frac{w}{8\pi} |Y(n\omega_0)| \left(\frac{w}{2\omega_0} \right)^{n-1} \ln \frac{\Lambda}{\omega_0}.$$

FIG. 4. I-V curves at different temperatures and spin bias. Voltage is measured in units of the characteristic potential w, and current in units of w/G_0. Dotted line: Ohm's law $I = 2G_0V$. Solid lines: $V_\sigma = 0$ and different temperatures $T = 0, 0.2w, 1.0w$ from bottom to top. Dashed lines: $T = 0$ and different values of spin bias $V_\sigma = 0.1w, 0.3w, 0.5w$ from bottom to top. The initial part of the I-V curves is shown in the inset

Consider now the case of the voltage bias when the system is driven by external voltage $V + V_1 \cos \omega t$, and assume the limit of large voltages, $V \approx \omega_0 \gg w$. In this case we can consider the second term in the left-hand-side of Eq. 73 as a small perturbation. If the ac voltage is absent we obtain the result coinciding with (77). Small ac voltage modifies I-V curves, and the most impressive part of this modification is the resonant steps analogous to the Shapiro steps in the Josephson junctions[37]. In contrast to Josephson junctions these steps are not at constant voltage, but at constant current $I = ef$ like in the regime of Coulomb blockade[38] and in the regime of sliding CDW in linear-chain conductors[21]. At this current the frequency of the ac voltage is equal to the frequency of current oscillations in the wire. The width of the step at $V \gg w$ and $V_1 \ll V$ can be calculated straightforwardly

using perturbative approach. We find with logarithmic accuracy

$$V_{step} = \frac{V_1 w}{\pi V}|F(\omega_0)| \ln \frac{\Lambda}{\omega_0}.$$

Non-zero spin bias induces a spin current which contains both dc and ac parts. The spin current can be calculated according to relation

$$I_\sigma = \frac{\sqrt{2}}{\pi}\partial_t\langle\hat{\Phi}_\sigma\rangle,$$

where $\hat{\Phi}_\sigma$ can be found from equation of motion (23) with $\sin\sqrt{2}\hat{\Phi}_\sigma$ which is calculated using equations (65) and (68). Then we obtain for spin current

$$I_\sigma = \frac{V_\sigma}{2\pi} + \frac{w}{2\pi}\cos\sqrt{2}\Phi_\rho(t)\int_0^\infty dt_1 \cos\sqrt{2}\Phi_\rho(t-t_1)\frac{T\sin V_\sigma t_1}{\sinh \pi T t_1}e^{-w\int_{t-t_1}^{t}\cos^2\sqrt{2}\Phi_\rho(t_2)dt_2}. \quad (79)$$

In the limit of large voltages, $\omega \gg w$ the spin current can be presented in a simple analytic form

$$I_\sigma = \frac{V_\sigma}{2\pi}\left(1 + \frac{w}{\pi\omega_0}\sin\omega_0 t\right).$$

V. NON-IDEAL CONTACT

A. Refermionization in the spin channel

As non-ideal contacts induce Friedel oscillations in the quantum wire, one can expect that such contacts must induce the effects in transport which are similar to those in the system with impurity studied in previous sections. This statement is supported by the results of our letter [33] where we have studied the spinless LL with two identical non-adiabatic contacts. However the problem of transport through non-adiabatic contact to a quantum wire with a spinful interacting electron gas was not solved. In this section we consider electronic transport through a clean quantum wire described as a spinful LL with one ideal adiabatic and the second non-ideal contact. The main difficulty in solving this problem is, again, large fluctuations of the displacement field $\hat{\Phi}_\sigma$. And, again, we solve this problem by means of refermionization in the spin channel.

To study the role of the non-ideal contacts we act similarly to the previous sections, we solve equation of motion for the displacement fields with boundary conditions.

Consider boundary conditions for the spin channel with $K_\sigma = 1$ with ideal adiabatic contact at $x = L$ and non-adiabatic contact at $x = 0$. Then boundary conditions read

$$(v_F \partial_x - \partial_t)\hat{\Phi}_\sigma(x = 0) = \hat{P}_\sigma^L - \sqrt{2} f \varepsilon_F \sin \sqrt{2}\hat{\Phi}_\sigma \cos \sqrt{2}\hat{\Phi}_\rho \tag{80}$$

$$(v_F \partial_x + \partial_t)\Phi_\sigma(x = L) = \hat{P}_\sigma^R.$$

As field $\hat{\Phi}_\sigma(x, t)$ satisfies the equation of motion

$$\left(v_F^2 \partial_x^2 - \partial_t^2\right)\hat{\Phi}_\sigma(t, x) = 0. \tag{81}$$

We can find solution for $\hat{\Phi}_\sigma(x, t)$ in terms of its values at the contacts, and using then boundary conditions (80) we can obtain equation of motion for the displacement field at the non-ideal contact

$$\partial_t \hat{\Phi}_\sigma + \sqrt{2} f \varepsilon_F \sin \sqrt{2}\hat{\Phi}_\sigma \cos \sqrt{2}\hat{\Phi}_\rho = \frac{1}{2}[\hat{P}_\sigma^L(t) - \hat{P}_\sigma^R(t - t_L)]. \tag{82}$$

This equation resembles equation of motion for the displacement field at the impurity site. In order to map the problem of non-ideal contact to the impurity problem we consider the model LL with ideal contacts at $x = \pm L$ and an impurity characterized by back-scattering matrix element $\tilde{W}$ at $x = 0$. Then equation of motion for the displacement field and boundary conditions for such an impurity read

$$\left(v_F^2 \partial_x^2 - \partial_t^2\right)\hat{\Phi}_\sigma(t, x) = \sqrt{2} v_F \tilde{W} \sin \sqrt{2}\hat{\Phi}_\sigma \cos \sqrt{2}\hat{\Phi}_\rho \delta(x), \tag{83}$$

$$(v_F \partial_x \mp \partial_t)\hat{\Phi}_\sigma(x = \mp L) = \hat{Q}^{L,R}.$$

Here we denote external sources of fluctuations as $\hat{Q}$, and later we will relate them to the source terms $\hat{P}$.

Next, similar to the case of contacts we derive equation of motion for the phase at the impurity site $x = 0$

$$\partial_t \hat{\Phi}_\sigma + \frac{\tilde{W}}{\sqrt{2}} \sin \sqrt{2}\hat{\Phi}_\sigma \cos \sqrt{2}\hat{\Phi}_\rho = \frac{1}{2}[\hat{Q}^L(t - t_L) - \hat{Q}^R(t - t_L)]. \tag{84}$$

Now we compare equations (82) and (84) and find that equations of motion become identical if we choose

$$\tilde{W} = 2 f \varepsilon_F, \quad \hat{Q}^L(t) = \hat{P}^L(t + t_L), \quad \hat{Q}^R(t) = \hat{P}_\sigma^R(t)$$

Thus using such substitutions we can use the results for spin channel obtained in Sec IV A to study the problem of electron transport in quantum wire with one non-ideal and the second ideal contacts.

B. Solution for the charge channel

Now let us consider the charge channel. Following the method used in Sec. II C we find solution of equation of motion for $\hat{\Phi}_\rho(x,\omega)$ satisfying the boundary conditions. In this way we obtain expression for the displacement field which depends on the values of both $\hat{\Phi}_\rho$ and $\hat{\Phi}_\sigma(x,\omega)$ at the boundary with non-adiabatic contact, as both these fields are present in the non-linear term of the boundary condition (17). Then using this solutions at $x = 0$ we find non-linear equations of motion for $\hat{\Phi}_\rho(x = 0)$ which are similar to (22), but with different memory function Z and different right-hand containing the source terms $P_{L,R}$ in a non-symmetric way

$$\partial_t \hat{\Phi}_\rho + f \varepsilon_F Z \otimes \sin \sqrt{2} \hat{\Phi}_\sigma \cos \sqrt{2} \hat{\Phi}_\rho = F_L \otimes \hat{P}_\rho^L - F_R \otimes \hat{P}_\rho^R. \tag{85}$$

For the short-range interaction Fourier components of the memory functions read

$$Z(\omega) = F_L(\omega) = K_\rho \frac{1 - i K_\rho \tan 2\omega t_L}{2 K_\rho - i(1 + K_\rho^2) \tan 2\omega t_L}, \quad F_R(\omega) = \frac{K_\rho}{2 K_\rho \cos 2\omega t_L - i(1 + K_\rho^2) \sin 2\omega t_L}.$$

In the limit of strong interaction between the electrons ($K_\rho \ll 1$) these equations will give the results similar to the case of impurity.

In case of long-range Coulomb interaction between the electrons we act as in Sec. II C and find

$$Z(\omega) = F_L(\omega) = \frac{i\omega Q_+ - \omega^2(Q_+^2 - Q_-^2)}{1 + 2i\omega Q_+ - \omega^2(Q_+^2 - Q_-^2)}, \quad F_R(\omega) = \frac{i\omega Q_-}{1 + 2i\omega Q_+ - \omega^2(Q_+^2 - Q_-^2)},$$

$$Q_\pm(\omega) = \frac{v_F}{L} \sum_{k=-\infty}^{\infty} \frac{(\pm 1)^k}{\omega^2 - q_k^2 v^2(q_k)}.$$

In limiting cases $\omega = 0$ and $\omega \gg v_F/L$ which is a proper limit for long enough wires, we find relations for memory functions similar to the case of the impurity

$$Z(\omega) = F_L(\omega) \approx \frac{1}{2\sqrt{\gamma \ln \frac{2v_F}{\omega d}}}, \quad F_R(\omega) \approx \frac{e^i q_\omega L}{2\sqrt{\gamma \ln \frac{2v_F}{\omega d}}}, \quad q_\omega = \frac{\omega}{v_F \sqrt{2\gamma \ln \frac{2v_F}{\omega d}}}.$$

where q_ω is solution of equation $\omega = q_\omega v(q_\omega)$.

Thus we find that the problem of electron transport in quantum wire with one non-ideal and the second ideal contacts is mapped to the impurity problem. Then all the results obtained in Sec. IV can be used for the case of non-ideal contacts if we substitute the impurity potential W for $f\varepsilon_F$ which characterizes the amplitude of the Friedel oscillations induced by the non-ideal contact.

VI. CONCLUSIONS

Using the approach based on the bosonised Tomonaga-Luttinger Hamiltonian we have studied electronic transport in 1D conductors with a single isolated impurity or with non-ideal contacts to leads of higher dimension, and predicted a new dynamical regime of conduction in which applied voltage in addition to dc-current induces ac oscillations with the wash-board frequency $f = \bar{I}/e$. This regime is different from the well-known regime of macroscopic tunneling leading to the power-law I-V curves and occurs in case of repulsive interaction between the electrons and is related to sliding of the Friedel oscillations induced by the defect.

We have considered both short-range interaction and long-range Coulomb interaction. The former appears in gated quantum wires where the long-range part of the interaction is screened by electrons of 3D gate electrodes.

In order to calculate I-V curves we have derived boundary conditions at the contacts which take into account that the current leads of higher dimension play a role of heat bath for the quantum wire.

Calculations of the transport properties are based on the equations of motion derived for the phase fields at the impurity site. The equations resemble equations of motion of a quantum pendulums. Therefore, there are solutions corresponding to rotation of the pendulum, that is solutions in which the phase increases linearly with time and the linear increase is accompanied by time-periodic oscillations. Since time derivative of the phase determines current in the wire, this corresponds to the regime of conductance in which dc current is accompanied by generation of ac current. The solution of the problem is complicated by strong fluctuations of the phase fields. For the case of spinless electrons we describe fluctuations under assumption that fluctuations are Gaussian. This assumption is justified in the limits of large voltages and/or strong interaction between electrons, otherwise it is a model assumption. In case of spinful interaction the problem becomes more complicated because in practically important cases K_σ is not small, therefore, fluctuations in the spin channel are neither small nor Gaussian. However, fluctuations in the spin channel can be treated strictly in case of spin-rotation invariant interaction, when interaction does not dependent on spin and $K_\sigma = 1$. Then we can use refermionization in the spin channel which makes the impurity Hamiltonian quadratic in spin variables. This enables us to solve the problem in

the limits of strong short-range interaction ($K_\rho \ll 1$) or moderate long-range interaction.

As non-adiabatic contacts between the quantum wire and leads of higher dimensions produce Friedel oscillations in the quantum wire, similar non-stationary effects in the electronic transport can be induced by non-ideal contacts.

We have found the dynamical regime of conduction when the applied voltage exceeds the threshold value. The magnitude of the threshold voltage is determined by $2k_F$-component of defect potential reduced by quantum fluctuations. Thermal fluctuations further reduce the threshold voltage, therefore, the effect of impurity on conductance is suppressed at temperatures $T > T_0 \sim V_T$. Furthermore, fluctuations destroy the effect of the impurity in relatively short wires, shorter than the length of the order of v/V_T. Thus the effect considered in our work can be observed at low enough temperature in a relatively long quantum wire with a single impurity or a non-ideal contact, with minimal length and maximal temperature being related to magnitude of impurity potential and the strength of inter-electronic repulsion.

The impurity potential W can be of different origin and of different strength. If the defect is induced by an impurity atom in the conduction channel then the potential can be quite large, of the order of the Fermi energy, but if the defect is made artificially, say, by a potential of a point contact, then W can be quite small. In semiconductor based quantum wires with shallow impurity W is expected to be of the order of few millivolts. In this case the range of frequencies of generated ac signal can be quite large, up to practically important teraherz region, depending on material of the quantum wire and the origin of the defect. We expect that one of the methods of experimental observation of current oscillations is to search for resonant Shapiro states in the presence of the applied ac-voltage.

We have also found that I-V curves of the quantum wire with defect depend on the spin bias applied to the system. Thus spin bias and spin current can be measured electrically. This is related to violation of the spin-charge separation at the impurity site. In the considered case of spin-rotation invariant interaction ($K_\sigma = 1$) the spin current cannot be induced by electric voltage if there is no spin bias. It is interesting to note, that in Gaussian model for spin channel one can obtain non-zero spin current as the result of spontaneous symmetry breaking, even in case when spin bias is absent. However, as the Gaussian model cannot be justified when K_σ is not small compared to 1, these results cannot be taken seriously. Nevertheless spin current can be induced in the absence of spin bias if the impurity is magnetic[39]. In this case the electric voltage induces not only the electrical current, but the spin

current as well.

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